STATISTICAL

STUDY

A STUDY OF CORRELATION BETWEEN IQ, APTITUDE TESTS IN MATHEMATICS AND QUEBEC PROVINCIAL HIGH SCHOOL SCORES IN MATHEMATICS

1. ABSTRACT

Essentially this paper tries to answer the question whether there is any correlation between success on IQ, PSAT and SAT tests and success on provincial high school final examinations in mathematics. Of particular interest are two subjects; elementary algebra and geometry, where alleged success in both subjects is supposedly guaranteed by virtue of high IQ, senior high PSAT and SAT scores.

Of interest also, is the alleged mathematical superiority of boys performance over that of girls-a myth that is held by a lot of teachers and indeed, by some students as well.

(1) Are we justified in assuming that all twenty(20) of the distribution scores, presented below, are normally distributed? If they are not, then all of the usual tests would not apply since most of the comparative tests are based on the fact that distributions sampled are normal. Fortunately, it is found that all distributions satisfy the chi^2 test for goodness of fit. The results are shown in Table 1 on page 31.

(2) Is there a significant difference between the results obtained for boys and that obtained for girls? There was no significant difference as found in the scores of IQ, PSAT, SAT or any subject in mathematics. A histogram comparison of the boys' scores vs the girls' scores on the ten(10) tests is shown on pages 10-20. The comparison is done by reflection about the x-axis.

Using (1) and (2) above, the ten test scores of both boys and girls were combined and the subsequent study carried out on the combined scores. However, there is a slight discrepancy since Table 1 indicates that using the Kolmogorov-Smirnov test for goodness of fit fails in one case-that of PSAT maths scores when the sexes are combined. The subsequent study was carried out anyway

under the assumption that the PSAT maths scores were normally distributed.

(3) What sort of linear correlation is obtained between certain subjects such as IQ and Geometry, for example? Can one find any high linear multiple correlations?

(4) If it is assumed that there is a correlation, what is it and how accurately can one predict success in geometry or elementary algebra knowing the IQ, PSAT and SAT scores?

Briefly, the results show that a small positive correlation is found between geometry, algebra and IQ, PSAT and SAT, but elementary algebra seems to be the only subject about which one might predict success. The possible reasons for this will be discussed under section 4. Also, 2,3,4,5,6 and 7 variable prediction equations(linear) for success in geometry, elementary algebra, trigonometry, intermediate algebra and analytic geometry were calculated and are presented with their respective coefficients of multiple correlation.

The IQ test scores used were from some form of the OTIS self-administering type(Alpha, Beta or Gamma) and in the case where there was more than one test administered, the average IQ was taken. The PSAT and SAT scores were percent rank relative to the high school scales presented by Princeton, New Jersey. The scores in mathematical subjects were Quebec Provincial Protestant examination results(scored out of 100%).

2. PROCEDURE

A. MODUS OPERANDI

1. Raw data was first separated into two subsets-girls and boys, and the data was then partitioned into the following categories:

(1) IQ
(2) PSAT(Verbal)
(3) PSAT(Maths)
(4) SAT(Verbal)
(5) SAT(Maths)
(6) Geometry
(7) Elementary Algebra
(8) Trigonometry
(9) Intermediate Algebra
(10) Analytic Geometry

The above ten(10) categories were symbolized in the following ways: (1) All girls' scores ended with a "G". (2) All boys' scores ended with a "B". (3) Combined scores of boys and girls have neither a "B" nor a "G" at the end. Thus, for example, the PSAT(Verbal) scores for boys would be symbolized by PSATVB, those for girls by PSATVG and the combined scores of both boys and girls would simply be PSATV. The shorthand notation for the ten(10) categories shown above are listed below:

(1) OTIS
(2) PSATV
(3) PSATM
(4) SATV
(5) SATM
(6) GEOM
(7) EALG
(8) TRIG
(9) IALG
(10) ANAL

The same order of categories is maintained throughout the paper to avoid confusion.

2. Raw scores in the above subjects were separated according to sex. The chi^2 "goodness-of-fit" test was applied to both sets of 10. It can be seen from Table 1 on page 31 that most of the sets of scores were not significant at the .05 level. That is, most of the values were found to be greater than .05. Of the few sets where the chi^2 values were found to be lower, a Kolmogorov-Smirnov test was applied. This was done for three reasons: (1) The measures of central tendency were very similar, as were the histograms. (2) The K-S test takes into account the full distribution rather than just the central tendencies. (3) No t-test applied was found to be significant when trying to find any differences between boys' and girls' scores on any of the tests.

3. Means, standard deviations and histograms were obtained for the ten(10) groups of boys' scores and compared with the ten(10) groups of girls' scores. t-tests were also made and the level of significance for all ten cases is summarized in Table 2 on page 32. The histograms found are reflected about the x-axis to illustrate how similar the various distributions are. Above each pair of histograms are shown the main statistical characteristics.

4. Forty-five(45) linear correlation coefficients between all pairs of combined(boys + girls) scores were found[${}^{10}C_2$] and are presented in Table 3 on page 33.

5. Also shown on pages 34-43 are histogram plots of the combined scores in each and every subject of both boys and girls. A normal curve is included in each plot to show how closely the particular subject scores approximate it. Atop each plot are the actual statistics calculated from the particular distribution including skewness and kurtosis.

6. Two variable linear regression lines of the form $y = ax + b$ were also found with standard errors for "a" and "b" along with 3-D histograms. The plots of the regression lines and 3-D histograms are shown in pages 44-128. Note that the regression line of "x" on "y" is derived with the histogram while that of "y" on "x" is derived with the plot of the linear regression line. Thus, prediction equations of two variables can me derived either way. Examples will follow.

7. Using the raw data 3, 4, 5, 6 and 7 variable linear regression prediction equations were obtained for all mathematical subjects along with the multiple correlation coefficients and standard errors. These results are shown in tables 4-14.

B. SUBJECTS

The subjects were all english speaking protestants from a predominantly rural community in Quebec. The data was obtained from the permanent high school records. As seen from the raw data at the end of this paper, not all pupils have taken all of the tests so that the population size varies for each correlation coefficient determined. A copy of the raw data(without names) is presented in the appendix.

C. TEST CONDITIONS

It can be safely assumed that there was a uniform testing situation for all tests given.

3. RESULTS

As pointed out above the results are presented in both tabular and graphic formats and there are several interesting observations gleaned from the tables 3.

(1), a higher correlation between IQ and PSAT and SAT scores than between these scores and the scores in high school mathematics which they are supposed to help predict. Evidently, whatever the PSAT and SAT tests are measuring, it seems to be more correlated to IQ scores than success in mathematics.

(2), a somewhat lower correlation coefficient between IQ and maths scores in general than one might suspect. Indeed, one might debate the fact that the correlation coefficient obtained for geometry versus IQ, for example, could be satisfactorily explained by the tacit assumption that since higher IQ pupils write provincial examinations anyway, there is bound to be some positive correlation. In any case, the only correlation values that look promising for prediction purposes should be values greater than .65 at the very least.

(3), it is found that success in analytic geometry is negatively correlated to success in the PSAT maths aptitude test! Granted that the sample is small, but it does engender some thought about the so-called maths aptitude test and the type of reasoning involved in analytic geometry.

(4), when we examine the overall multiple correlation coefficient for 3,4,5,6 and 7 variable regression equations, we find that except for elementary algebra, multiple correlation coefficients are not exceptionally high and there is not too much certainty in predicting scores.

4. DISCUSSION

Overall, one might say that the results are somewhat disappointing. Even though it might be argued that the samples were too small or did not cover the

complete spectrum of student types, it is still an indication that the notion of high IQ or high PSAT or SAT maths scores guaranteeing success in high school mathematics must be dispelled forever.

The above tables and graphs have to be perused carefully before any use can be made of them for prediction purposes. In certain instances it is seen that the multiple correlation coefficient is extremely high where the sample size is low. The coefficient in question would certainly decrease if the sample were to increase. Thus, one has to be careful before using these values for prediction.

It is apparent, however, that in the cases of elementary algebra a particular prediction can be made with a fair degree of certainty. One suspects that the cause of this is that the type of reasoning used in IQ, PSAT(Maths) and SAT(Maths) tests is predominantly that of abstraction, symbol substitution and pattern recognition similar to that used in elementary algebra. Geometry, however, where one uses intuition and deduction or intermediate algebra where these processes are coupled with induction, spatial relations and sometimes model or game type of reasoning, the IQ, PSAT(Maths) and SAT(Maths) do not seem to involve enough of this type of reasoning. Indeed, these tests involve very little set type reasoning processes.

However, there is one interesting observation to be gleaned from the 3-D histograms. It seems that in almost every case of two variable correlation, it is found that scores in the 65th to 80th percentiles support the block indicating the highest frequency. Thus, for example, in the case of IQ vs geometry, using this reasoning one would expect to find that IQ scores ranging from about 115 to 125 would be most highly correlated with scores in geometry(or algebra or whatever) in the range of 75% to 85%. This suggests a sub-study culling out only those individuals that fall into this category. All the correlation coefficients would increase, it is certain, but then, all one is saying is that a "B" student will do "B" work no matter what subject he studies. It seems that the A, C, D, and F students are the most erratic, for some reason.

In any case the teacher, educator or counsellor will have to use the equations for predicting success in a particular subject very carefully-making certain first that the multiple correlation coefficient is high enough to warrant use of the equation and always bearing in mind that the equation itself is just a guide a certain percentage of the time.

5. RECOMMENDATIONS

(A) The chief difficulties with this study were that the samples in certain instances were not large enough and that the data did not sufficiently represent a large enough cross-section of society as a whole-i.e., students from all socioeconomic groups. Hence, a more comprehensive undertaking should be made with perhaps some other criteria added, such as students' knowledge of games, success in arithmetic at lower levels etc.

(B) It would be interesting to investigate the reliability coefficients for the aptitude tests and the provincial exams as it was tacitly assumed that these

remain relatively constant throughout the years. Also, what about the validity of the provincial exams and the aptitude tests? Once these questions are satisfactorily answered, then some more enlightening statistical studies might be done on the influence of mathematics teachers in the classroom.

6. CALCULATIONS

The first order of business is to make certain that the sets of scores for both boys and girls do not differ significantly. That is, if it is found that all sets of scores for boys is essentially normally distributed as well as all sets of scores for girls, then by the <u>Central Limit Theorem</u> the combined sets of scores will also be normally distributed. The advantage of this, of course, is that it increases the number of scores in each subject. The greater the population, the more accurate the statistics found therein. The chi² test was deployed for comparison to a normal distribution. The formula used is:

$$\chi^2 = \sum_{i=1}^{k} \left[\frac{(o_i - e_i)^2}{e_i} \right] \qquad (1)$$

where "o" represents the observed or actual values and "e" represents the expected or theoretically correct values calculated from the well-known formula for the normal curve.

The equation of the normal curve is given by,

$$y = \frac{1}{s\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-m}{s}\right)^2} \qquad (2)$$

Here "m" is the mean and "s" the standard deviation. It is seen in the first formula above, that if there is a perfect match between observed and theoretical values, chi² will equal 0. On the other hand, if the observed data deviates a lot from the expected values, chi² will be quite large. The trick is to determine whether the derived value of chi² is too large or not relative to the given distribution with which one is making a comparison. A continuous distribution has been derived for just such a purpose. From this distribution, a chi² statistic is derived, whose value lies between 0 and 1 on the chi² distribution curve. Different curves are derived for their respective "degrees of freedom". The advantage of calculating the value on the chi² distribution curve is obvious. Hopefully, the value obtained will be as close to 1 as possible. The nearer the value to 1, the better the fit. Conversely, if values derived are less than .05, it means that the distribution in question is not a good fit at all to the known, compared curve.

The formula for the chi² distribution curve is given by,

$$f(\chi^2) = \frac{1}{2^{\frac{\nu}{2}}\Gamma\left(\frac{\nu}{2}\right)}(\chi^2)^{\frac{\nu-2}{2}} e^{-\frac{x^2}{2}} \tag{3}$$

where "nu" represents the "degrees of freedom" and capital "GAMMA"(Γ) signifies the gamma or factorial function.

Recall that the gamma function is given by the expression,

$$\begin{aligned} \Gamma(n+1) &= \int_0^\infty x^n e^{-x} dx \\ \Gamma(n) &= \int_0^\infty X^{n-1} e^{-x} dx \end{aligned} \tag{4}$$

From these formulas and by applying some elementary calculus we obtain the following interesting relations:

$$\begin{aligned} \Gamma(n+1) &= n\Gamma(n) \\ \Gamma(n+1) &= n! \\ \Gamma\left(\frac{1}{2}\right) &= \sqrt{\pi} \end{aligned} \tag{5}$$

The Kolmogorov-Smirnov(K-S) tests were also deployed for goodness of fit. These tests are more accurate for they take into account all of the aspects of the distribution to be tested, rather than focusing on the measures of central tendency only. Analogous formulas to those shown above yield more accurate "goodness of fit" statistics and for that reason, these results were adopted rather than the regular chi² test.

Once the normality of all sets of scores was established, a t-test, using independent means was deployed. First, a formula is used to calculate the t-statistic. Then this calculated value is converted by means of a t-distribution into a value from 0 to 1, as was done in the chi² test. Usually, one looks for a 95% significance level. This means that the distribution value of t cannot be less than .05. Again, we hope that the values of the t-statistic calculated from the distribution will be well in excess of the .05 value.

It might prove instructive to examine both formulas for calculating the t-statistic. First we use the following formula,

$$t_{ind} = \frac{m_x - m_y}{\left(\frac{1}{n_x} + \frac{1}{n_y}\right)^{\frac{1}{2}} \left[\frac{\sum x_i^2 - n_x m_x^2 + \sum y_i^2 - n_y m_y^2}{n_x + n_y - 2}\right]^{\frac{1}{2}}} \tag{6}$$

where m, s and n denote the mean, standard deviation and number of cases of the two samples respectively.

Next, by means of the following formula, t-values are found along the distribution curve given by,

$$f(t) = c\left[1+\frac{t^2}{\nu}\right]^{-\frac{\nu+1}{2}} \tag{7}$$

where c is a constant, t is the value obtained from the previous formula and nu represents the degrees of freedom.

An equivalent formula where the constant "c" is evaluated is given by,

$$f(t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)\left[1+\frac{t^2}{\nu}\right]^{\frac{\nu+1}{2}}} \tag{8}$$

All calculations done in this paper were done on an IBM AT computer using the STATGRAPHICS program, version 3.0. Although all the formulas presented in this section are theoretically correct, series approximations are used in most cases to the desired degree of accuracy-in this case, four places.

The easiest way to find measures of central tendency is to calculate moments about the origin and the mean.

The "m" values which are primed are moments taken about the origin whereas the "m" values without primes are moments taken about the mean "m". Hopefully, there will be no confusion with the m's since the mean used in this paper never appears with a numerical subscript. The value of a_3 is known as the "skewness" and the value of a_4 is known as the "kurtosis." For the normal curve the skewness is obviously zero because of symmetry and the kurtosis works out to have the value 3.

The summary statistics shown are all based on the well-known statistical formulas which are presented below:

$$m_k' = \frac{\sum_{i=1}^{n} x_i^k}{n}; \quad m_k = \frac{\sum_{i=1}^{n} [x_i - m]^k}{n}$$
$$a_3 = \frac{m_3}{(m_2)^{\frac{3}{2}}}; \quad a_4 = \frac{m_4}{(m_2)^2} \tag{9}$$

A short method of calculating moments about the origin involves finding the "moment generating function" of a distribution. This is found from the formula,

$$M_g(\theta) = \sum_{x=1}^{n} e^{\theta g(x)} P(x) \tag{10}$$

where the left hand side represents the moment generating function and P(x) is the probability distribution function for discrete data. The formula holds as well for continuous data except that an integral takes the place of the summation symbol.

To generate moments about the origin, the following formula is used,

$$M_k' = \left.\frac{d^k M}{d\theta^k}\right|_{\theta=0} \tag{11}$$

where k derivatives are taken and evaluated at $\theta=0$. A simple example might prove instructive.

The binomial distribution has,

$$P(x) = {}^nC_x p^x q^{n-x}$$

$$\begin{aligned} M_x(\theta) &= \sum_{i=1}^{n} e^{\theta x} \frac{n!}{x!\,(n-x)!} p^x q^{n-x} \\ &= \sum_{i=1}^{n} \frac{n!}{x!\,(n-x)!} \left(pe^{\theta} \right)^x q^{n-x} \\ M_x(\theta) &= \left(q+pe^{\theta}\right)^n \end{aligned} \tag{12}$$

Using differentiation according to the above formula, we have,

$$\begin{aligned} M_x'(\theta) &= npe^{\theta}(q+pe^{\theta})^{n-1} \\ M_x''(\theta) &= npe^{\theta}(q+pe^{\theta})^{n-2}(q+npe^{\theta}) \end{aligned} \tag{13}$$

Now letting $\theta=0$ and realizing that $p+q=1$(by definition of the binomial distribution), we obtain values for the first and second moments about the origin.

From these it is then easy to derive values for the mean and standard deviation as follows,

$$\begin{aligned} m_1' &= np; \quad m_2' = npq+n^2p^2 \\ m &= m_1' = np; \quad s^2 = m_2' - m_1'^2 \\ s^2 &= npq+n^2p^2-(np)^2 = npq \\ \therefore s &= \sqrt{npq} \end{aligned} \tag{14}$$

Recall that the general formula which relates moments about the origin and

moments about the mean is given by,

$$m_k = m_k' - k m_{k-1}' m_1' + \frac{k(k-1)}{2!} m_{k-2}' m_1'^2 - \cdots + (-1)^k m_1'^k \tag{15}$$

It might be instructive to recall some of the formulas for the summary statistics which are presented. The median or "middlemost" case is found by ordering the data and then observing which value lies exactly in the middle of the remaining data. The mode is determined by that value which occurs most often after ordering it. The geometric mean is given by the formula,

$$\left(\prod_{i=1}^{n} x_i\right)^{\frac{1}{n}} \tag{16}$$

and the standard error is given by,

$$S.E. = \frac{s}{\sqrt{n}} \tag{17}$$

where s is the standard deviation of a sample.

The next order of business is to explore the ways in which coefficients of correlation were calculated. First, consider two sets of data where only those individuals that obtained scores in both sets are used. Next, suppose a plot is made of each and every point (x,y) where each "x" score and "y" score denotes the two scores made by a particular individual. A "scatter" diagram would result with the number of points corresponding to the number of cases initially taken.

Now if one score could be exactly predicted knowing the other score(the ideal case), then a straight line could be used as a "nomograph" and all one would have to do is to take the OTIS IQ test, for example, and he or she would immediately know what their score would be in Elementary Algebra. Needless to say, the ideal case does not hold in real situations.

Still, more than likely, there might be a linear trend and of course, it is the statistician's duty to determine what straight line could best represent the set of points in question. The method of "least squares" is deployed for this purpose.

This method supposes that each point lies on some line and we would have a series of lines such that,

$$
\begin{aligned}
y_1 &= a+bx_1 \\
y_2 &= a+bx_2 \\
\cdot \quad & \quad \cdot \quad \cdot \\
\cdot \quad & \quad \cdot \quad \cdot \\
\cdot \quad & \quad \cdot \quad \cdot \\
y_n &= a+bx_n
\end{aligned}
\tag{18}
$$

It is seen that the a's and b's have to be the same if a single line of best fit is to be found. Since any given equation above is in general not the true equation that represents each and every point, there will be a slight difference from the "true" value.

These differences can be expressed as,

$$
\begin{aligned}
\Delta_1 &= y_1-(a+bx_1) \\
\Delta_2 &= y_2-(a+bx_2) \\
\cdot \quad & \quad \cdot \quad \cdot \\
\cdot \quad & \quad \cdot \quad \cdot \\
\cdot \quad & \quad \cdot \quad \cdot \\
\Delta_n &= y_n-(a+bx_n)
\end{aligned}
\tag{19}
$$

The method of least squares requires that the squares of these differences be a minimum. We can readily find what values of "a" and "b" will make the square of these differences a minimum by considering these differences as a function of a and b. That is, F(a,b) has to be a minimum.

We therefore have,

$$
\begin{aligned}
F(a,b) &= \sum_{i=1}^{n} \Delta_i^2 \\
&= [y_1-(a+bx_1)]^2+[y_2-(a+bx_2)]^2 + \ldots +[y_n-(x+bx_n)]^2 \\
&= \sum_{i=1}^{n} [y_i-(a+bx_i)]^2 \\
\frac{\partial F}{\partial a} &= -2\sum_{i=1}^{n} [y_i-(a+bx_i)] = 0 \\
\frac{\partial F}{\partial b} &= -2b\sum_{i=1}^{n} [y_i-(a+bx_i)] = 0
\end{aligned}
\tag{20}
$$

From the two partial derivative equations we can quickly solve to obtain

$$na + \left(\sum_{i=1}^{n} x_i\right)b = \sum_{i=1}^{n} y_i$$
$$\left(\sum_{i=1}^{n} x_i\right)a + \left(\sum_{i=1}^{n} x_i^2\right)b = \sum_{i=1}^{n} x_i y_i \qquad (21)$$

Dropping the limits of the sums for easier notation and solving these equations we obtain,

$$a = \frac{N_a}{D}; \qquad b = \frac{N_b}{D}$$
$$D = \begin{vmatrix} n & \sum x \\ \sum x & \sum x^2 \end{vmatrix}; \; N_a = \begin{vmatrix} \sum y & \sum x \\ \sum xy & \sum x^2 \end{vmatrix}; \; N_b = \begin{vmatrix} n & \sum y \\ \sum x & \sum xy \end{vmatrix} \qquad (22)$$

These results yield the desired values for "a" and "b" which are:

$$a = \frac{\sum y \sum x^2 - \sum x \sum xy}{n\sum x^2 - (\sum x)^2};$$
$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} \qquad (23)$$

Recall that the linear regression equation was solved for y in terms of x. The regression line can also be expressed as x in terms of y. That is, an equation of the form x=a'+b'y could be found. To find the solution for a' and b', it is only necessary to substitute x for y and y for x in the above expressions. This follows quickly since both are linear functions. If we do this, we have,

$$a' = \frac{\sum x \sum y^2 - \sum x \sum y}{n\sum y^2 - (\sum y)^2};$$
$$b' = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - (\sum y)^2} \qquad (24)$$

In the ideal case bb'=1 since the lines are essentially the same. The product bb' is called the "coefficient of determination" and if one imagines the situation geometrically, it is found that in the ideal case, the two straight lines will coincide. As the coefficient of determination decreases the two lines will intersect at a greater and greater angle. Recall that b and b' are the slopes of the given lines, in any case.

The correlation coefficient is taken as the square root of the coefficient of determination and is usually designated as r. Hence $r^2 = bb'$ and we obtain the formula,

$$r = \pm \frac{n\sum xy - (\sum x)(\sum y)}{\sqrt{\left[n\sum x^2 - (\sum x)^2\right]\left[n\sum y^2 - (\sum y)^2\right]}} = \frac{\sum (x-m_x)(y-m_y)}{ns_x s_y} \tag{25}$$

Multiple linear correlation coefficients are found in an analogous way. The method of least squares is again deployed. For three sets of data, there will be three coefficients, for four, there will be four and so on. The multiple correlation coefficient is found by solving these equations.

First, consider that one of the sets of variables is dependent on the others so that we might have,

$$\begin{aligned} X_1' &= c_0 + c_2X_2 + c_3X_3 + \ldots + c_kX_k \\ \text{Let } x_i &= X_i - X_{i_m}; \quad x_i' = X_i' - X_{i_m} \\ x_i' &= a_0 + a_2x_2 + a_3x_3 + \ldots + a_kx_k \end{aligned} \tag{26}$$

It is clear that by minimizing we get,

$$\sum \left(X_1 - X_1'\right)^2 = \sum \left(x_1 - x_1'\right)^2 \tag{27}$$

We can then express the differences as before in the following manner,

$$G(a_0, a_2, \ldots, a_k) = \sum \left[x_1 - a_0 - a_2x_2 - \ldots - a_kx_k\right]^2 \tag{28}$$

Taking partials and dividing by 2, we obtain,

$$\begin{aligned} &a_0n + a_2\sum x_2 + \ldots + a_k\sum x_k = \sum x_1 \\ &a_0\sum x_2 + a_2\sum x_2^2 + \ldots + a_k\sum x_2x_k = \sum x_2x_1 \\ &\quad\vdots \\ &a_0\sum x_k + a_2\sum x_kx_2 + \ldots + a_k\sum x_k^2 = \sum x_kx_1 \end{aligned} \tag{29}$$

Since $\Sigma x_i = 0$, $a_0 = 0$ and we are left with one less equation-the top one shown above. Recalling equation (25) shown above connecting the sums of differences from the mean with the correlation coefficient we have,

$$\sum x_i x_j = n r_{ij} s_i s_j \tag{30}$$

Substituting in the above system we obtain,

$$\begin{aligned}
a_2 r_{22} s_2 + a_3 r_{23} s_3 + \ldots + a_k r_{2k} s_k &= r_{21} s_1 \\
a_2 r_{32} s_2 + a_3 r_{33} s_3 + \ldots + a_k r_{3k} s_k &= r_{31} s_1 \\
\vdots \qquad \vdots \qquad\qquad \vdots \quad &\quad \vdots \\
a_2 r_{k2} s_2 + a_3 r_{k3} s_3 + \ldots + a_k r_{kk} s_k &= r_{k1} s_1
\end{aligned} \tag{31}$$

By the method of Cramer's Rule, it can be shown that the solution to the above equations becomes,

$$\frac{R_{11}}{s_1} x_1' + \frac{R_{12}}{s_2} x_2 + \frac{R_{13}}{s_3} x_3 + \ldots + \frac{R_{1k}}{S_k} x_k = 0 \tag{32}$$

where R_{ij} is the cofactor of the determinant R where the ith row and jth column has been struck out.

The determinant R is given by,

$$R = \begin{vmatrix} r_{11} & r_{12} & r_{13} & \ldots & r_{1k} \\ r_{21} & r_{22} & r_{23} & \ldots & r_{2k} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ r_{k1} & r_{k2} & r_{k3} & \ldots & r_{kk} \end{vmatrix} \tag{33}$$

An illustrative example might serve to clarify this method. However, it must be remembered that the means, standard deviations and correlation coefficients must be derived from only those cases that are relevant i.e the intersection of the sets of scores.

Unfortunately, the linear correlation coefficient tables do not yield values of r for any arbitrary number of cases. We must choose three sets for our example that would most closely approximate the actual values. For this, I have taken the three values of r with the largest number of cases. Hopefully, the actual means, SD's and r's will be a good enough approximation.

Needless to say, the STATGRAPHICS program picks out only those cases that are common to all three sets and calculates all means, SD's and r's based on that. So, the example shown is only a guide to how the calculation is performed, but with any luck, we should closely approximate the actual result found by the computer program!

Let us choose the three sets of scores for Geometry, Otis and Elementary

Algebra in that order so that r_{12} represents the correlation coefficient between Geometry and Otis, for example. From the various tables, we find,

SUBJECT	MEAN	STD DEVIATION	CORR COEFF
GEOM	63.69	13.31	r_{12} = .473
OTIS	111.84	8.93	r_{13} = .549
EALG	68.4	13.2	r_{23} = .513

We form the determinant,

$$R = \begin{vmatrix} 1 & .473 & .549 \\ .473 & 1 & .513 \\ .549 & .513 & 1 \end{vmatrix} = .24$$

$$R_{11} = (1)(1) - (.513)^2 = .737 \tag{34}$$

$$R_{12} = (-1)[(.473)(1) - (.543)(.549)] = -.191$$

$$R_{13} = (.473)(.513) - (.519)(1) = -.306$$

Substituting these values into the multiple correlation coefficient equation, we obtain,

$$\frac{.737}{13.31}x_1{}' - \frac{.191}{8.93}x_2 - \frac{.306}{13.2}x_3 = 0$$

$$\therefore .0554x_1{}' - .0214x_2 - .0232x_3 = 0$$

$$\therefore x_1{}' = .386x_2 + .419x_3 \tag{35}$$

$$\therefore X_1{}' - 63.69 = .383(X_2 - 111.84) - .423(X_3 - 68.4)$$

$$X_1{}' = .383X_2 + .423X_3 - 8.078$$

These results differ slightly from the true values of the coefficients given in Table 4. There, the x_2 coefficient was .3668 instead of our value of .383. The table value of the x_3 coefficient was .4144 whereas our approximate value was found to be .423. The constant coefficient also differs slightly. The table value is -5.2 whereas our value is -8.078. It might be noted that only slight variations in any one of the coefficients affects the other values considerably.

We recall the formula for the multiple correlation coefficient,

$$r_{1 \cdot 23 \ldots k} = \sqrt{1 - \frac{R}{R_{11}}}$$

$$r1 \cdot 23 = \sqrt{1 - \frac{.24}{.737}} = \sqrt{.673} = .821 \tag{36}$$

The value of the multiple correlation coefficient squared differs significantly from the value in Table 4. Thus, one can see how much the correlation coefficient is affected by the number of cases studied.

The last order of business is to investigate just how programs like STATGRAPHICS calculates the actual values of some of the statistical entities. As one can readily see, most of the above formulas are straightforward enough and the past bouts of tedium with these formulas is a thing of the past, thanks to the modern microcomputer.

However, some of the areas of the probability curves have to use series approximations. The chi² distribution, t-distribution and F distribution areas all have to use series. Recall that the F distribution is used in conjunction with the distribution of standard deviations. The ratio of two S.D.'s is considered and compared to known distributions with certain degrees of freedom not unlike those of the t and chi² distributions.

The area under the normal curve can be directly calculated from the formula given in this paper. Sometimes the following approximation is used:

$$P(x) = 1 - r(a_1 t + a_2 t^2 + a_3 t^3) + \epsilon(x)$$

$$a_1 = .4361836$$

$$a_2 = -.1201676$$

$$a_3 = .9372980$$

where:

$$r = \frac{e^{\frac{x^2}{2}}}{\sqrt{\pi}} \tag{37}$$

$$t = \frac{1}{1 + .3326x}$$

$$|\epsilon(x)| < 10^{-6}$$

For the chi² distribution the calculation is slightly more complicated. The "tail end" values are given by two sets of formulas. One is used if nu is even, the other is used if nu is odd:

$$
\nu odd: \quad t.e.v. = \frac{1-(\chi^2)^{\frac{\nu+1}{2}} e^{-\frac{x^2}{2}}}{1 \cdot 3 \cdot 5 \ldots \nu} \cdot \left(\frac{2}{\chi^2 \pi}\right)^{\frac{1}{2}} \cdot z
$$

$$
\nu even: \quad t.e.v. = 1 - \frac{(\chi^2)^{\frac{\nu}{2}} e^{-\frac{x^2}{2}}}{2 \cdot 4 \cdot 6 \ldots \nu} \cdot z \qquad (38)
$$

$$
z = 1 + \sum_{m=1}^{\infty} \frac{(\chi^2)^m}{(\nu+2)\cdot(\nu+4)\ldots(\nu+2m)}
$$

For the F distribution, it gets even more involved for we have two degrees of freedom to consider-nu$_1$ and nu$_2$. We divide the problem into three cases. Case 1, where n$_1$ is even(nu$_2$ can be even or odd).

For this case, we have:

$$
Q(F|\nu_1,\nu_2) = \left[1 + \frac{n}{2}(1-x) + \frac{n(n+2)}{2 \cdot 4}(1-x)^2 + \frac{n(n+2)(n+4)}{2 \cdot 4 \cdot 6}(1-x)^3 + \ldots\right] [x^{\frac{n}{2}}]
$$

For case 2, we have nu$_2$ even(nu$_1$ can be even or odd). This function is given by,

$$
Q(F|\nu_1,\nu_2) = \left[1 + \frac{n}{2}x + \frac{n(n+2)}{2 \cdot 4}x^2 + \frac{n(n+2)(n+4)}{2 \cdot 4 \cdot 6}x^3 + \ldots\right][1-(1-x)^{\frac{n}{2}}]
$$

In both of these formulas, we have,

$$
\nu_1 = n, \quad x = \frac{\nu_2}{\nu_2 + \nu_1 F}, \quad n = 1,2,3,\ldots \qquad (41)
$$

The third and last case is where both nu$_1$ and nu$_2$ are odd:

$$
Q(F|\nu_1,\nu_2) = 1 - A(t|\nu_2) + B(\nu_1,\nu_2)
$$

$$
A(t|\nu_1,\nu_2) = \frac{2}{\pi}\left(\theta + \sin\theta\cos\theta\left[1 + \frac{2}{3}\cos^2\theta + \frac{2 \cdot 4}{3 \cdot 5}\cos^4\theta + \frac{2 \cdot 4 \cdot 6}{3 \cdot 5 \cdot 7}\cos^6\theta + \ldots\right]\right)
$$

$$
\theta = \tan^{-1}\sqrt{\frac{\nu_1 F}{\nu_2}}\ ; \quad A(t|\nu_1,\nu_2) = \frac{2\theta}{\pi} \quad for\ \nu_2 = 1
$$

Note that this expression is the same as the expansion for the t distribution(shown below) except that nu$_2$ is odd! We now define the beta function and associated formulas,

$$\beta(\nu_1,\nu_2) = \frac{2}{\sqrt{\pi}}\frac{\left(\frac{\nu_2-1}{2}\right)!}{\left(\frac{\nu_2-2}{2}\right)!}\sin\theta\cos^n\theta\left(1+\frac{n+1}{3}\sin^2\theta+\ldots+\frac{m+n-4}{3\cdot 5\cdots(n-2)}\cos^{m-3}\theta\right)$$

where $m = 3,5,7,\ldots$ *and* $n = 1,3,5,7,\ldots$

$\beta(1,n) = 0$, $\beta(m,n) = 0$, *If* $m = 1$ *and* $n = 1$, *cofficient is* $\frac{2}{\pi}$

Also, $\theta = \tan^{-1}\sqrt{\frac{mF}{n}}$

We can also write different expansions for the above coefficient, viz,

$$\frac{2^{\frac{n+1}{2}}\left(\frac{n-1}{2}\right)!}{1\cdot 3\cdot 5\ldots(n-2)\pi} = \frac{2^{n-1}\left(\frac{n-1}{2}\right)!\left(\frac{n-3}{2}\right)!}{(n-2)!\pi} = \frac{2}{\sqrt{\pi}}\frac{\left(\frac{n-1}{2}\right)!}{\Gamma\left(\frac{n}{2}\right)} \tag{44}$$

We recall that,

$$\begin{aligned}
\Gamma\left(\tfrac{1}{2}\right) &= \sqrt{\pi} \\
\Gamma\left(\tfrac{3}{2}\right) &= \tfrac{1}{2}\Gamma\left(\tfrac{1}{2}\right) = \tfrac{1}{2}\sqrt{\pi} \\
\Gamma\left(\tfrac{5}{2}\right) &= \tfrac{3}{2}\Gamma\left(\tfrac{3}{2}\right) = \tfrac{3}{2}\tfrac{1}{2}\sqrt{\pi} \\
&\;\;\vdots \\
\Gamma\left(\tfrac{n}{2}\right) &= \frac{1\cdot 3\cdot 5\cdots}{2\cdot 2\cdot 2\cdots}\sqrt{\pi}
\end{aligned} \tag{45}$$

To evaluate the beta functions, we must remember that,

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} \tag{46}$$

We derive immediately from this that,

$$\begin{aligned}
B(m,1) &= \frac{2}{\pi} \\
B(m,3) &= \frac{2}{\pi}\frac{1!}{\frac{1}{2}} \\
B(m,5) &= \frac{2}{\pi}\frac{2!}{\frac{1}{2}\cdot\frac{3}{2}} \\
B(m,7) &= \frac{2}{\pi}\frac{3!}{\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{5}{2}} \\
&\vdots
\end{aligned} \tag{47}$$

Finally, we show the series used in evaluating the area under the t distribution except for the two tails. Again, the calculation must be done using two cases. The first case for finding the area of the t distribution is given by,

$$A\left(\frac{t}{n}\right) = \frac{2}{\pi}\left(\theta + \sin\theta\left[\cos\theta + \frac{2}{1\cdot 3}\cos^3\theta + \frac{2\cdot 4}{1\cdot 3\cdot 5}\cos^5\theta + \frac{2\cdot 4\cdot 6}{1\cdot 3\cdot 5\cdot 7}\cos^7\theta + \ldots\right]\right)$$

$$\textit{when } n = 3,5,7,\ldots$$

$$\textit{Also when } n = 3,5,7,\ldots \textit{ the series } 1 + \frac{2}{1\cdot 3} + \frac{2\cdot 4}{1\cdot 3\cdot 5} + \frac{2\cdot 4\cdot 6}{1\cdot 3\cdot 5\cdot 7} + \ldots$$

$$\textit{may be rewritten: } \frac{2^{n-3}\left(\frac{[n-3]!}{2}\right)^2}{(n-2)!}$$

$$\textit{When } n = 1, \quad A\left(\frac{t}{n}\right) = \frac{2\theta}{\pi}$$

The second case for finding the t distribution area is obtained from the following expression using even values of n,

$$A\left(\frac{t}{n}\right) = \sin\theta\left(1 + \frac{1}{2}\cos^2\theta + \frac{1\cdot 3}{2\cdot 4}\cos^4\theta + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cos^6\theta + \ldots\right)$$

$$\textit{The series } 1 + \frac{1}{2} + \frac{1\cdot 3}{2\cdot 4} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6} + \ldots = \frac{(n-2)!}{2^{n-2}\left[\left(\frac{n-2}{2}\right)!\right]^2} \tag{49}$$

$$\textit{In the above two series } \theta = \arctan\left(\frac{t}{\sqrt{n}}\right)$$

Remember that the probability for 2 tails is given by 1 - A(t/n) whereas

the probability for 1 tail is given by [1-A(t/n)] ÷ 2. Also, the integral form of the t distribution area is given by,

$$f(t) = c\int_0^{\infty} v^n e^{-\frac{v^2}{2}\left(1+\frac{t^2}{n}\right)} dv \tag{50}$$

The following examples illustrate the use of the results obtained in this paper by using the above mathematical techniques:

Example 1: Find the possible score in geometry of a student whose IQ is 120.

Answer: From the results on page 53, we have that,
GEOM = .704(OTIS)-14.605
= .704(120)-14.605 = 69.875.
The standard error is 11.72. Thus, the student has a 22.4% likelihood of obtaining a geometry mark of 69.875±11.72 i.e. between 58.155 and 81.395.

Example 2: What score in trigonometry might we expect from a student who obtains a score of 90% on the PSAT maths test, a 70% on the SAT maths test, a 75% in elementary algebra and who has an IQ of 105?

Answer: From the results in Table 6 on page 136, we have:
TRIG = .1392(OTIS)-.0082(PSATM)+.0442(SATM)+.3619(EALG)+22.9263.
= .1392(105) -.0082(90) + .0442(70) + .3619(75) +22.9263.
= 67.0408. The S.E. = 11.8228, hence there is a 19.41% chance that the student's mark in trigonometry will be anywhere between 55.218 and 78.8636.

The calculated values, graphs and tables computed make up the rest of this paper.

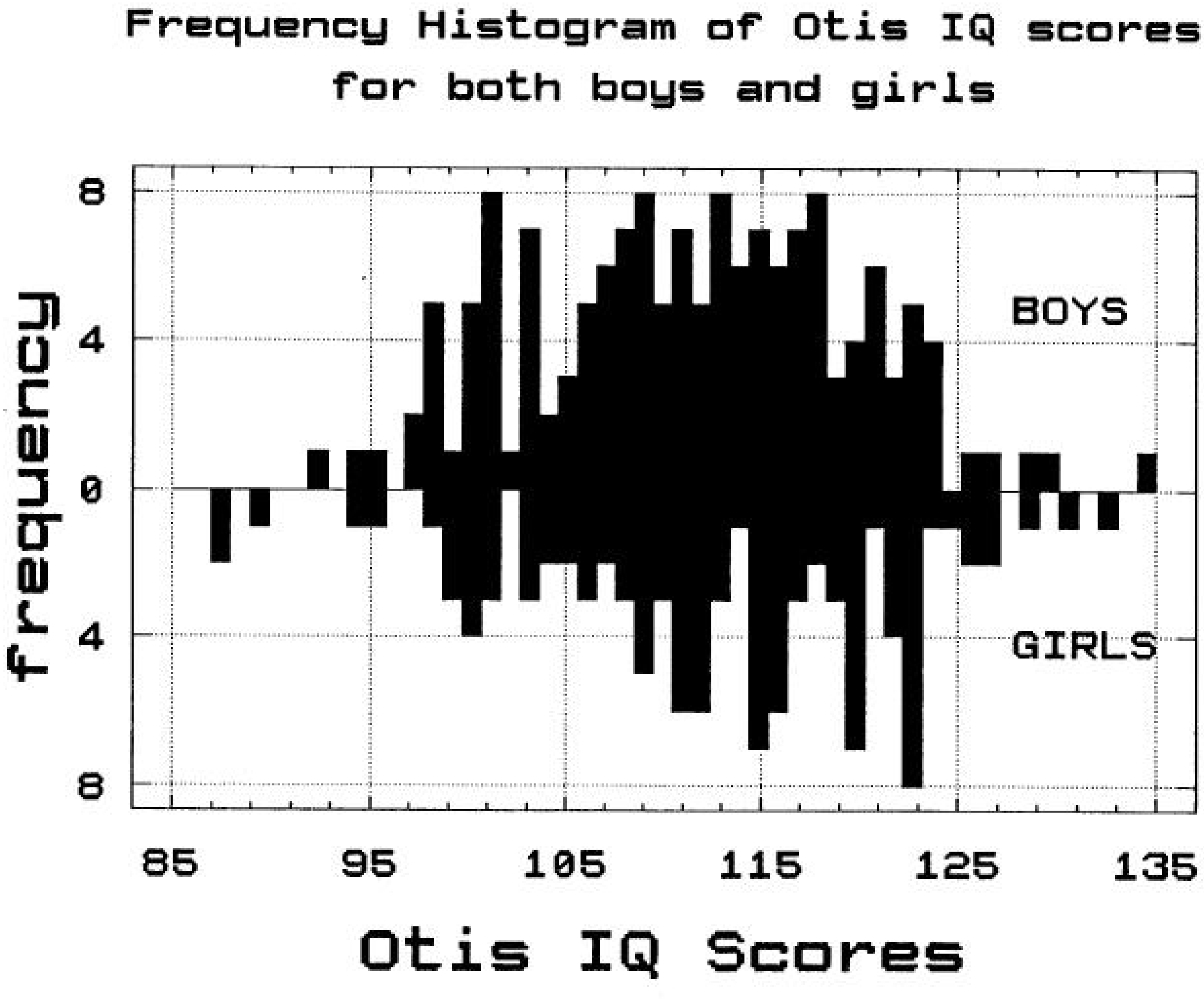
Two-Sample Analysis Results

		OTISB	OTISG	Pooled
Sample Statistics:	Number of Obs.	152	106	258
	Average	111.474	112.358	111.837
	Variance	68.3967	96.0988	79.7589
	Std. Deviation	[illegible]	9.803	[illegible]
	Median	112	113	112

```
Difference between Means = -0.884806
Conf. Interval For Diff. in Means:       95    Percent
 (Equal Vars.)      Sample 1 - Sample 2   -3.11082 1.3412     256 D.F.
 (Unequal Vars.)    Sample 1 - Sample 2   -3.18197 1.41236  200.7 D.F.

Ratio of Variances = 0.711732
Conf. Interval for Ratio of Variances:   0     Percent
                    Sample 1 - Sample 2

Hypothesis Test for H0: Diff = 0          Computed t statistic = -0.782931
                        vs Alt: NE        Sig. Level = 0.434391
                     at Alpha = 0.05      so do not reject H0.
```



Two-Sample Analysis Results

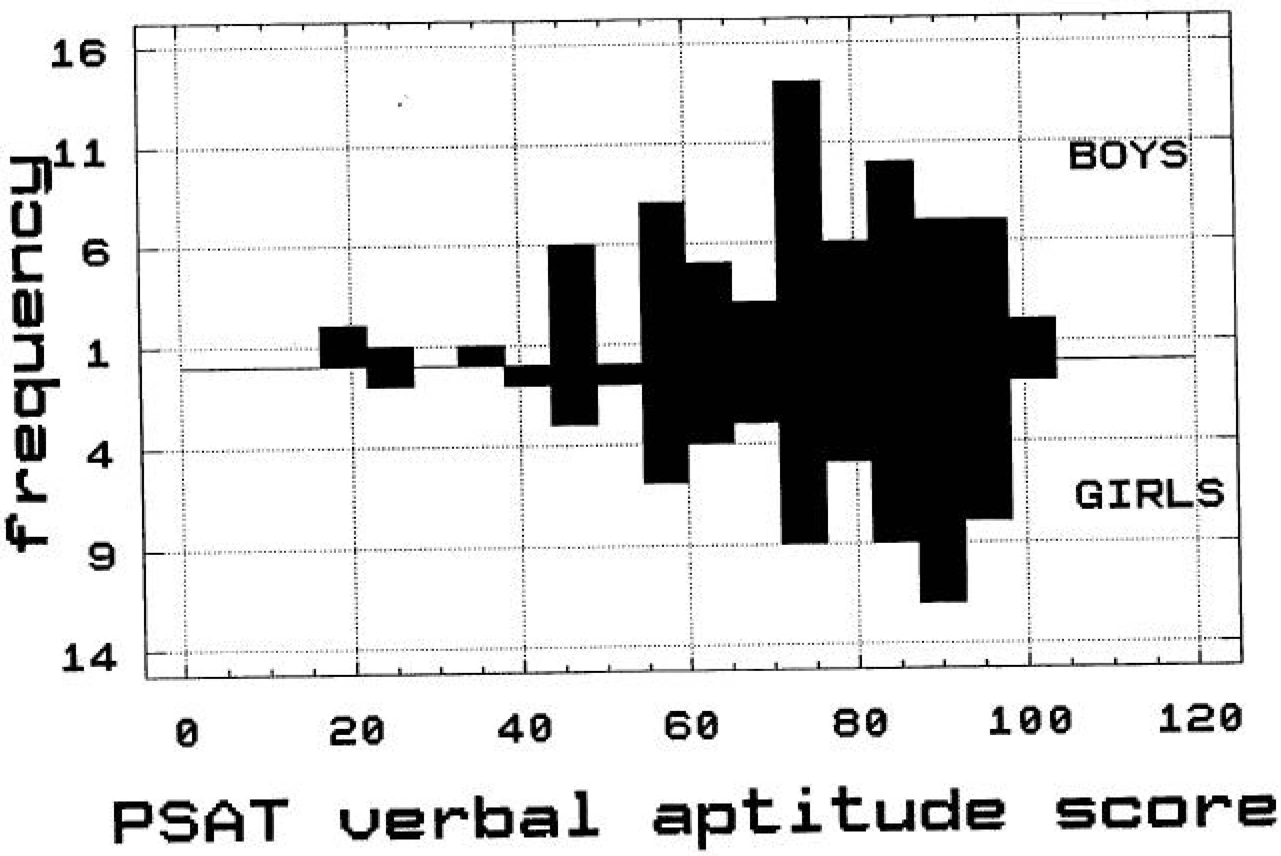
```
------------------------------------------------------------------------------------------
                                          PSATVB         PSATVG         Pooled
ample Statistics: Number of Obs.          72             63             135
                  Average                 72.6111        76.3333        74.3481
                  Variance                347.818        268.71         310.941
                  Std. Deviation          18.6499        16.3924        17.6335
                  Median                  76             80             76

ifference between Means = -3.72222
onf. Interval For Diff. in Means:         95     Percent
 (Equal Vars.)     Sample 1 - Sample 2    -9.74066 2.29622     133 D.F.
 (Unequal Vars.)   Sample 1 - Sample 2    -9.68902 2.24457   133.0 D.F.

atio of Variances = 1.2944
onf. Interval for Ratio of Variances:     0      Percent
                   Sample 1 - Sample 2

ypothesis Test for H0: Diff = 0           Computed t statistic = -1.22358
                        vs Alt: NE        Sig. Level = 0.223273
                     at Alpha = 0.05      so do not reject H0.
```

Frequency Histogram of PSAT verbal aptitude scores for both boys and girls



Two-Sample Analysis Results

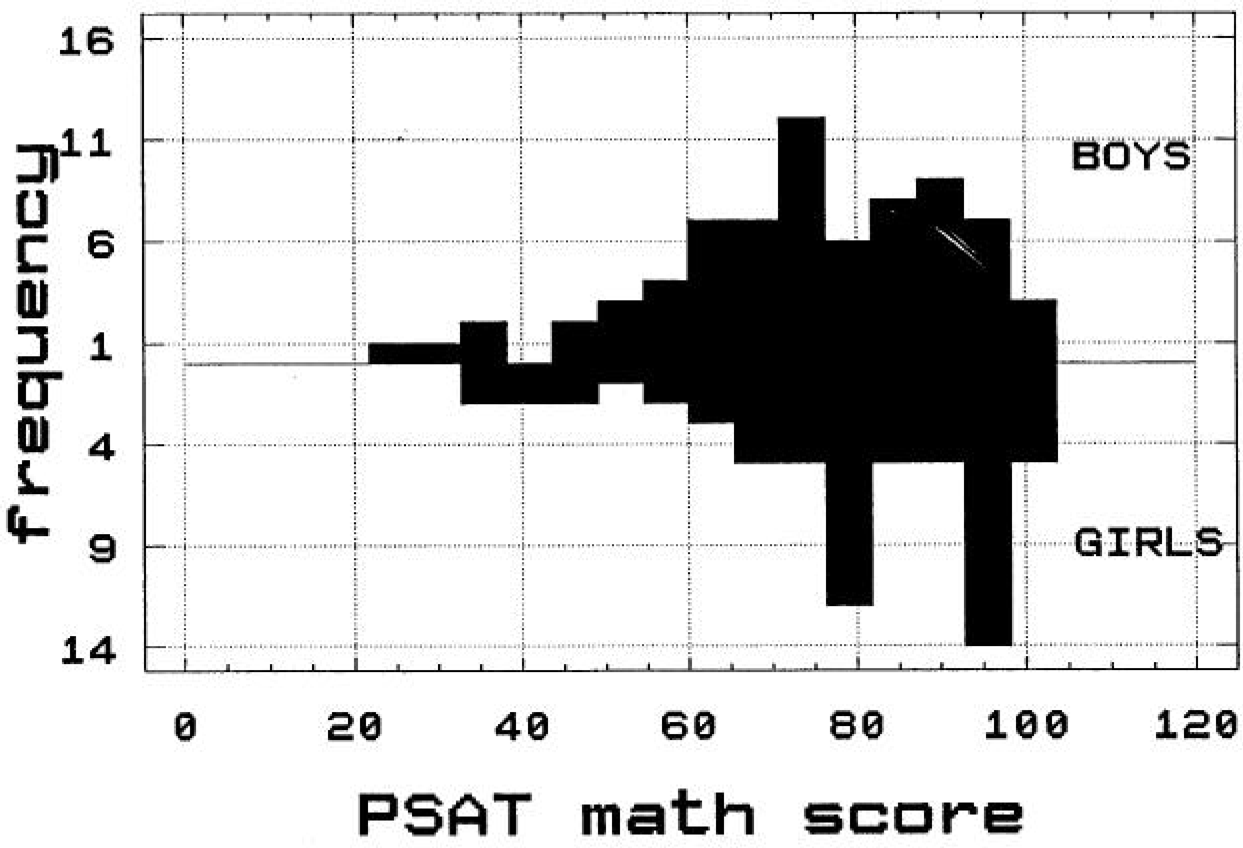
		PSATMB	PSATMG	Pooled
Sample Statistics:	Number of Obs.	72	63	135
	Average	74.5278	78.746	76.4963
	Variance	283.239	290.612	286.676
	Std. Deviation	16.8297	17.0473	16.9318
	Median	76	79	78

Difference between Means = -4.21825
Conf. Interval For Diff. in Means: 95 Percent
(Equal Vars.) Sample 1 - Sample 2 -9.99709 1.56058 133 D.F.
(Unequal Vars.) Sample 1 - Sample 2 -10.0032 1.56673 130.2 D.F.

Ratio of Variances = 0.974629
Conf. Interval for Ratio of Variances: 0 Percent
Sample 1 - Sample 2

Hypothesis Test for H0: Diff = 0 — Computed t statistic = -1.44413
vs Alt: NE — Sig. Level = 0.151053
at Alpha = 0.05 — so do not reject H0.

Frequency Histogram of PSAT mathematical aptutude scores for both boys and girls



Two-Sample Analysis Results

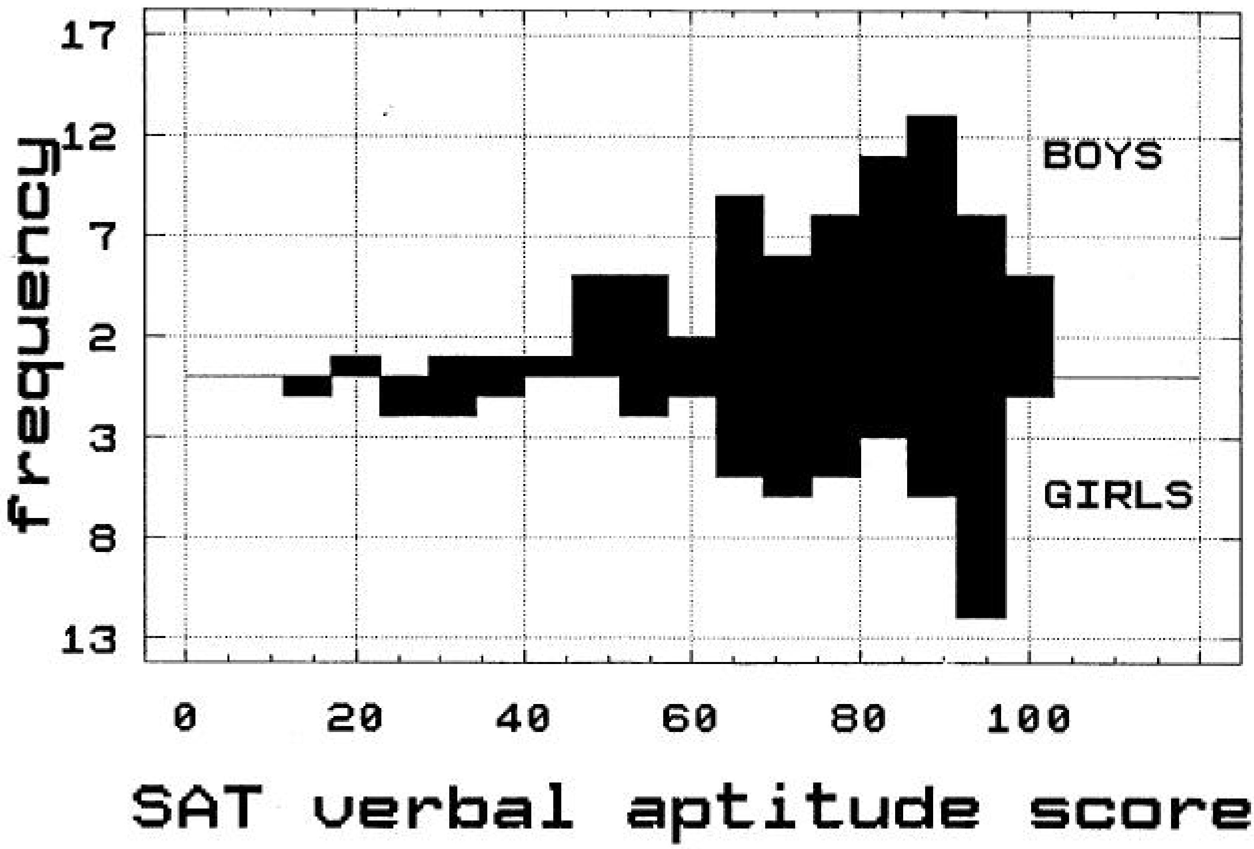
```
------------------------------------------------------------------------------------
                                               SATVB        SATVG        Pooled
Sample Statistics: Number of Obs.              76           47           123
                   Average                     75.6974      74.383       75.1951
                   Variance                    313.387      460.024      369.133
                   Std. Deviation              17.7027      21.4482      19.2128
                   Median                      80           78           80

Difference between Means = 1.31439
Conf. Interval For Diff. in Means:             95    Percent
 (Equal Vars.)     Sample 1 - Sample 2         -5.74552 8.3743      121 D.F.
 (Unequal Vars.)   Sample 1 - Sample 2         -6.10461 8.73339     83.8 D.F.

Ratio of Variances = 0.681241
Conf. Interval for Ratio of Variances:         0     Percent
                   Sample 1 - Sample 2

Hypothesis Test for H0: Diff = 0               Computed t statistic = 0.368668
                        vs Alt: NE             Sig. Level = 0.713019
                     at Alpha = 0.05           so do not reject H0.
```

Frequency Histogram of SAT verbal aptitude scores for both boys and girls



Two-Sample Analysis Results

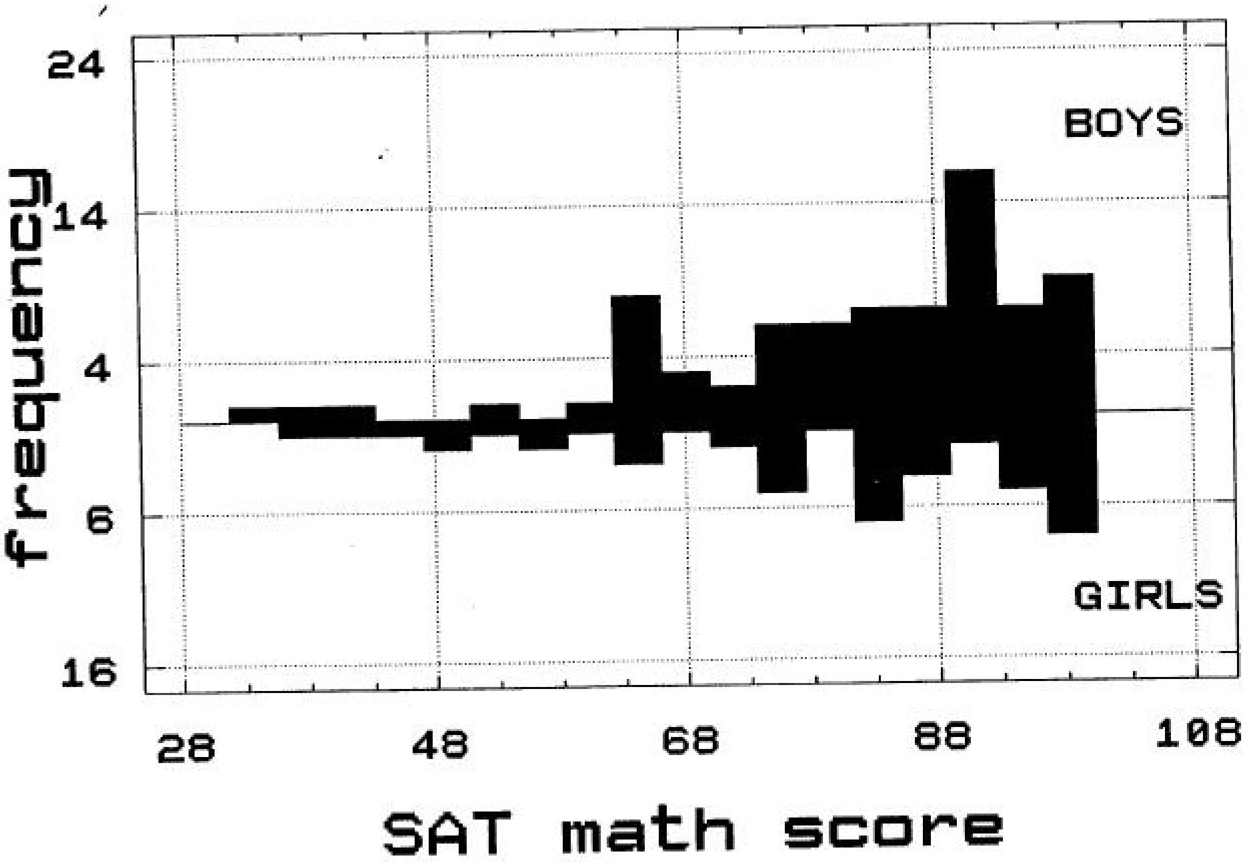
```
------------------------------------------------------------------------------------------
                                            SATMB         SATMG         Pooled
Sample Statistics: Number of Obs.           76            47            123
                   Average                  81.4342       78.1064       80.1626
                   Variance                 218.462       298.228       248.786
                   Std. Deviation           14.7805       17.2693       15.773
                   Median                   86            84            84

Difference between Means = 3.32783
Conf. Interval For Diff. in Means:          95     Percent
 (Equal Vars.)     Sample 1 - Sample 2      -2.46807 9.12372     121 D.F.
 (Unequal Vars.)   Sample 1 - Sample 2      -2.70945 9.3651    86.3 D.F.

Ratio of Variances = 0.732536
Conf. Interval for Ratio of Variances:      0      Percent
                   Sample 1 - Sample 2

Hypothesis Test for H0: Diff = 0            Computed t statistic = 1.13697
                     vs Alt: NE             Sig. Level = 0.257795
                  at Alpha = 0.05           so do not reject H0.
```

Frequency Histogram of SAT mathematical aptitude scores for both boys and girls



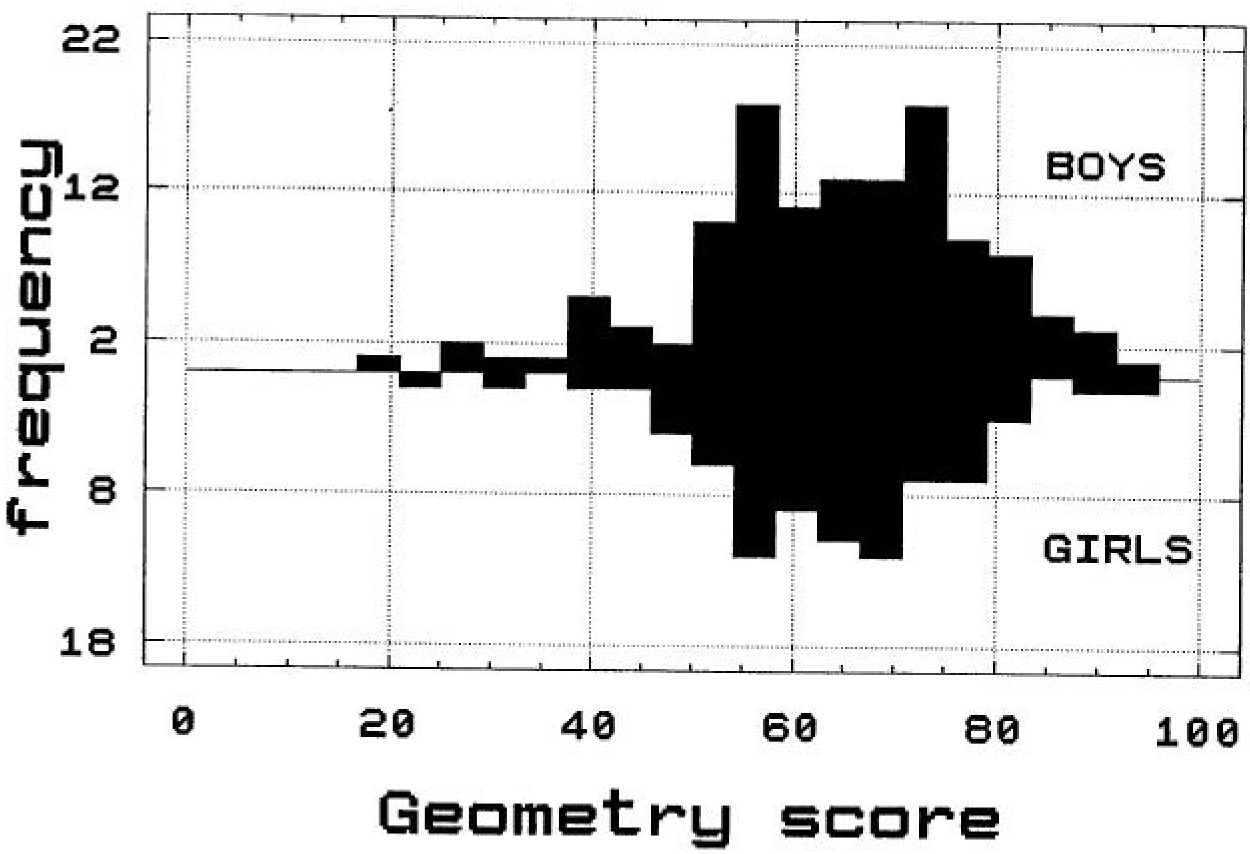
```
                           Two-Sample Analysis Results
--------------------------------------------------------------------------------
                                           GEOMB          GEOMG          Pooled
Sample Statistics: Number of Obs.          123            77             200
                   Average                 63.9675        63.2468        63.69
                   Variance                199.655        141.188        177.213
                   Std. Deviation          14.1299        11.8823        13.3121
                   Median                  65             63             64.5

Difference between Means = 0.720726
Conf. Interval For Diff. in Means:         95    Percent
 (Equal Vars.)      Sample 1 - Sample 2    -3.09495 4.53641     198 D.F.
 (Unequal Vars.)    Sample 1 - Sample 2    -2.94862 4.39007   181.5 D.F.

Ratio of Variances = 1.4141
Conf. Interval for Ratio of Variances:     0     Percent
                    Sample 1 - Sample 2

Hypothesis Test for H0: Diff = 0           Computed t statistic = 0.372568
                       vs Alt: NE          Sig. Level = 0.709868
                     at Alpha = 0.05       so do not reject H0.
```

Frequency Histogram of scores in high school geometry for both boys and girls



Two-Sample Analysis Results

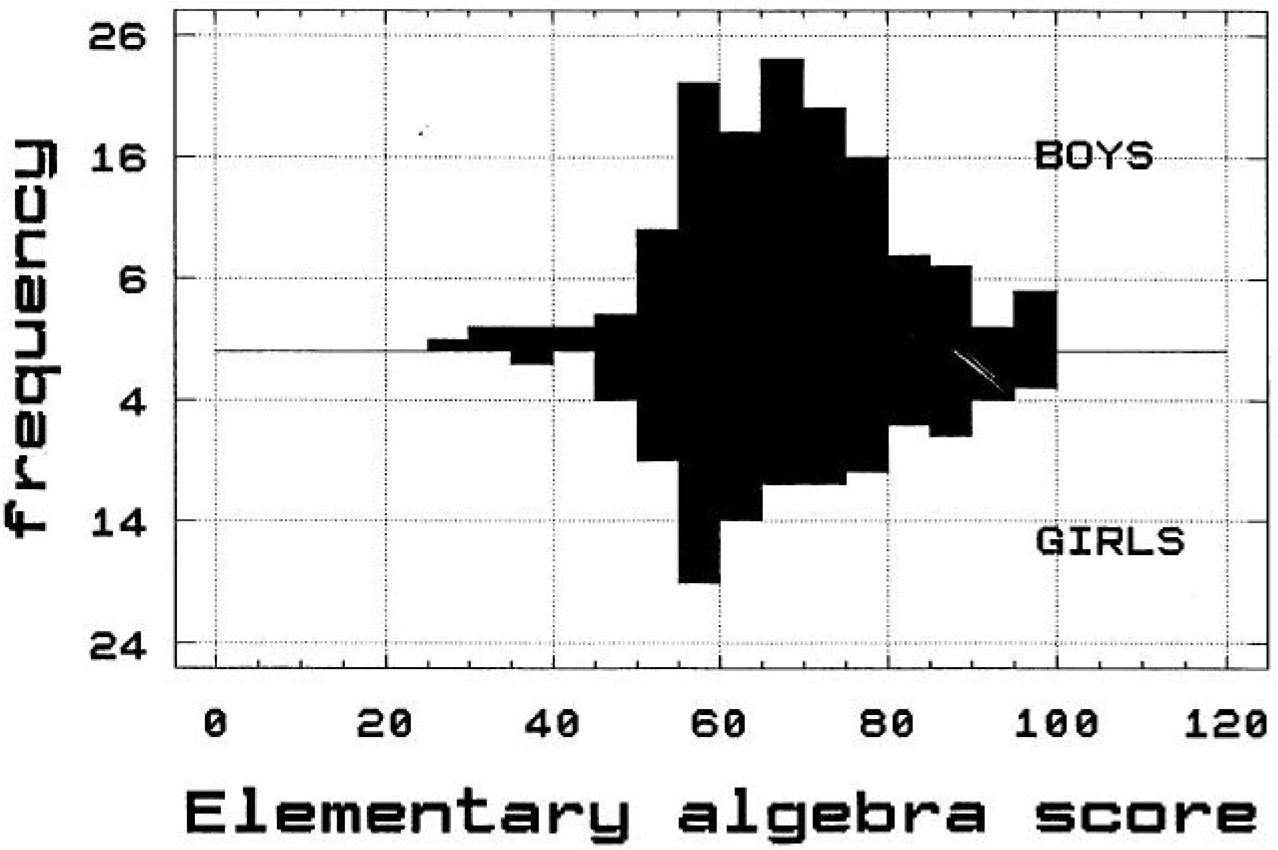
		EALGB	EALGG	Pooled
Sample Statistics:	Number of Obs.	142	99	241
	Average	67.8592	69.1414	68.3859
	Variance	176.023	171.796	174.29
	Std. Deviation	13.2673	13.1071	13.2019
	Median	68	68	68

```
Difference between Means = -1.28226
Conf. Interval For Diff. in Means:        95     Percent
 (Equal Vars.)     Sample 1 - Sample 2    -4.68816 2.12364     239 D.F.
 (Unequal Vars.)   Sample 1 - Sample 2    -4.6829 2.11838   212.6 D.F.

Ratio of Variances = 1.0246
Conf. Interval for Ratio of Variances:    0      Percent
                   Sample 1 - Sample 2

Hypothesis Test for H0: Diff = 0          Computed t statistic = -0.741811
                    vs Alt: NE            Sig. Level = 0.45893
                  at Alpha = 0.05         so do not reject H0.
```

Frequency Histogram of scores in high school algebra for both boys and girls



Two-Sample Analysis Results

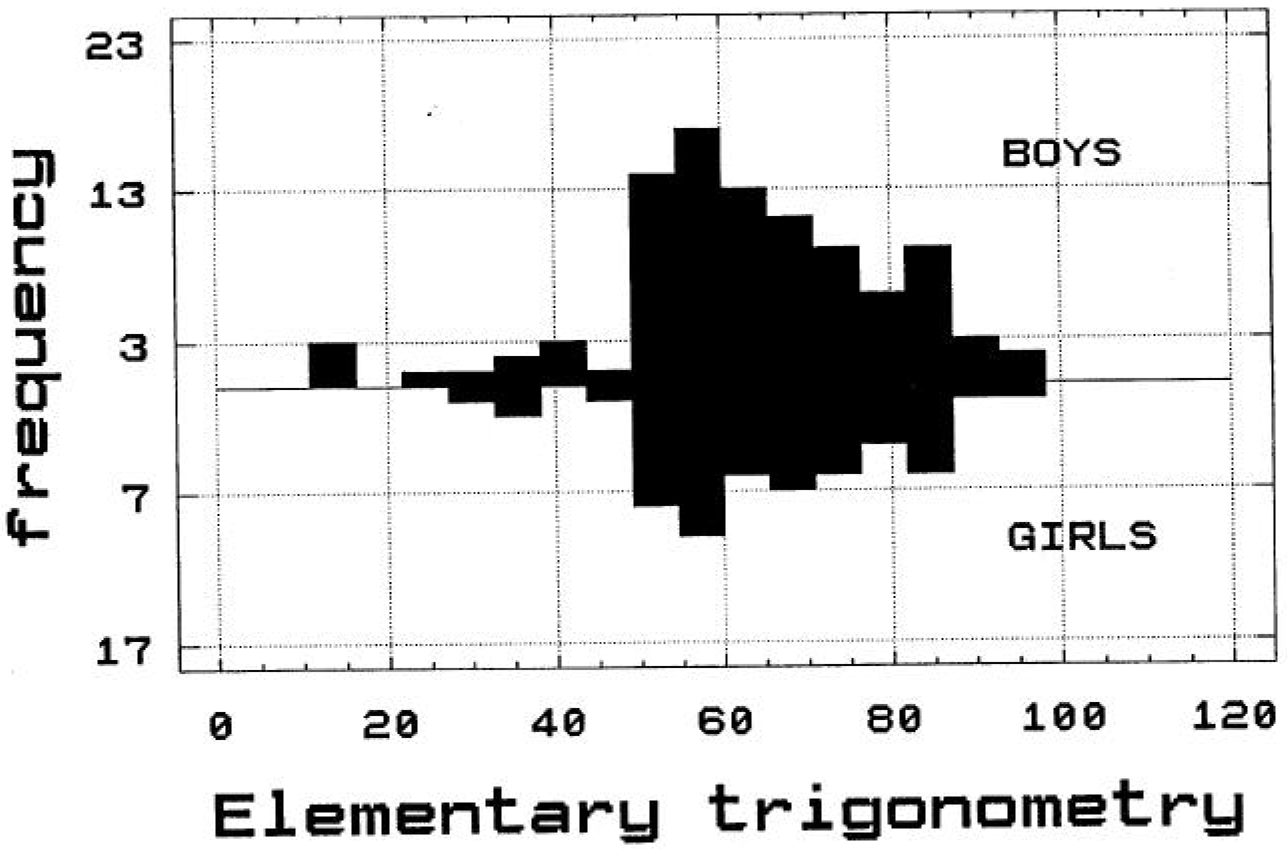
```
-------------------------------------------------------------------------------
                                        TRIGB        TRIGG        Pooled
Sample Statistics: Number of Obs.       95           53           148
                   Average              62.8737      64.9057      63.6014
                   Variance             281.388      215.779      258.021
                   Std. Deviation       16.7746      14.6894      16.063
                   Median               63           64           63

Difference between Means = -2.03198
Conf. Interval For Diff. in Means:      95    Percent
 (Equal Vars.)     Sample 1 - Sample 2  -7.47597 3.41202    146 D.F.
 (Unequal Vars.)   Sample 1 - Sample 2  -7.28398 3.22002  120.0 D.F.

Ratio of Variances = 1.30405
Conf. Interval for Ratio of Variances:  0     Percent
                   Sample 1 - Sample 2

Hypothesis Test for H0: Diff = 0        Computed t statistic = -0.737837
                        vs Alt: NE      Sig. Level = 0.461798
                     at Alpha = 0.05    so do not reject H0.
```

Frequency Histogram of scores in high school trigonometry for boys and girls



Two-Sample Analysis Results

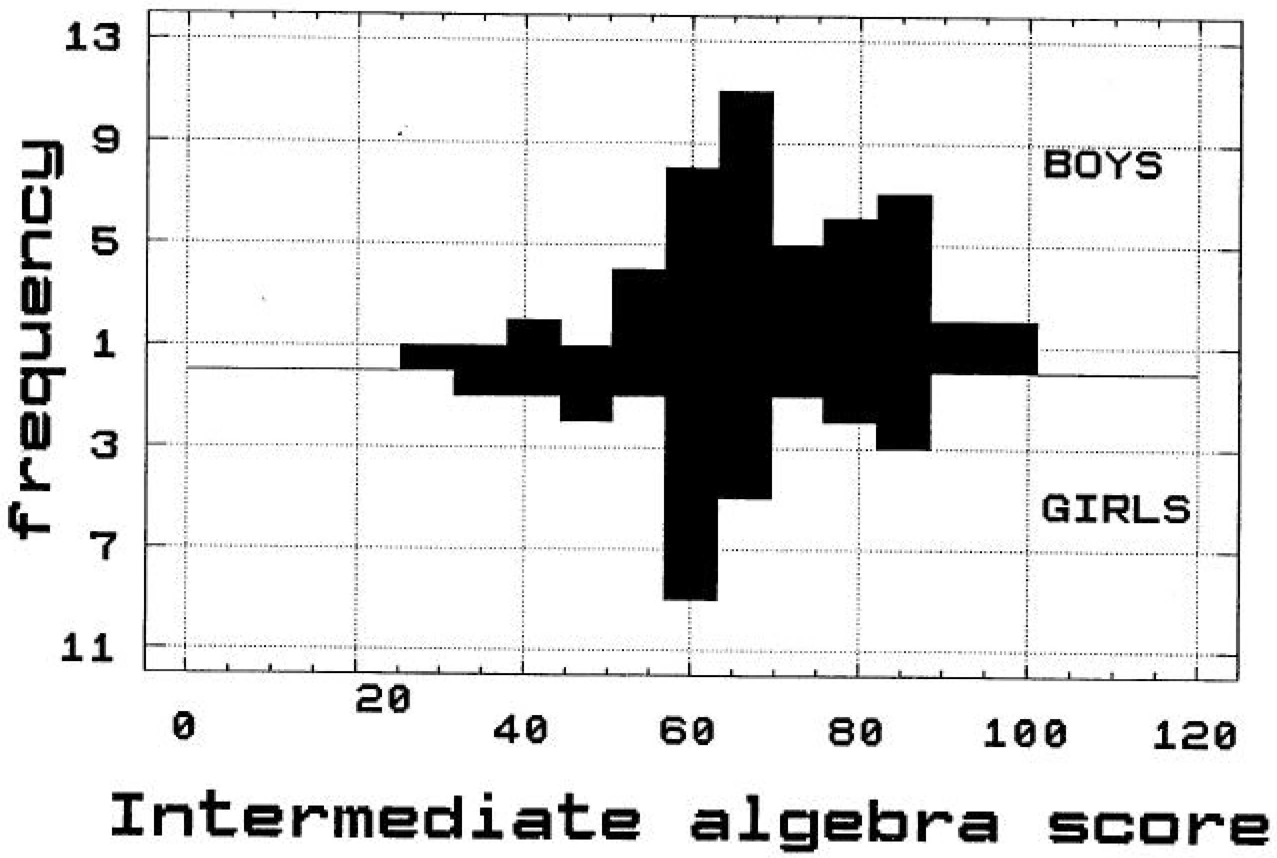
		IALGB	IALGG	Pooled
Sample Statistics:	Number of Obs.	50	25	75
	Average	68.64	63.32	66.8667
	Variance	227.868	181.977	212.78
	Std. Deviation	15.0953	13.4899	14.587
	Median	68	62	66

```
Difference between Means = 5.32
Conf. Interval For Diff. in Means:        95     Percent
 (Equal Vars.)     Sample 1 - Sample 2    -1.80272 12.4427      73 D.F.
 (Unequal Vars.)   Sample 1 - Sample 2    -1.58144 12.2214    53.2 D.F.

Ratio of Variances = 1.25218
Conf. Interval for Ratio of Variances:    0      Percent
                   Sample 1 - Sample 2

Hypothesis Test for H0: Diff = 0          Computed t statistic = 1.48892
                     vs Alt: NE           Sig. Level = 0.140817
                  at Alpha = 0.05         so do not reject H0.
```

Frequency Histogram of intermediate high school algebra scores for boys and girls

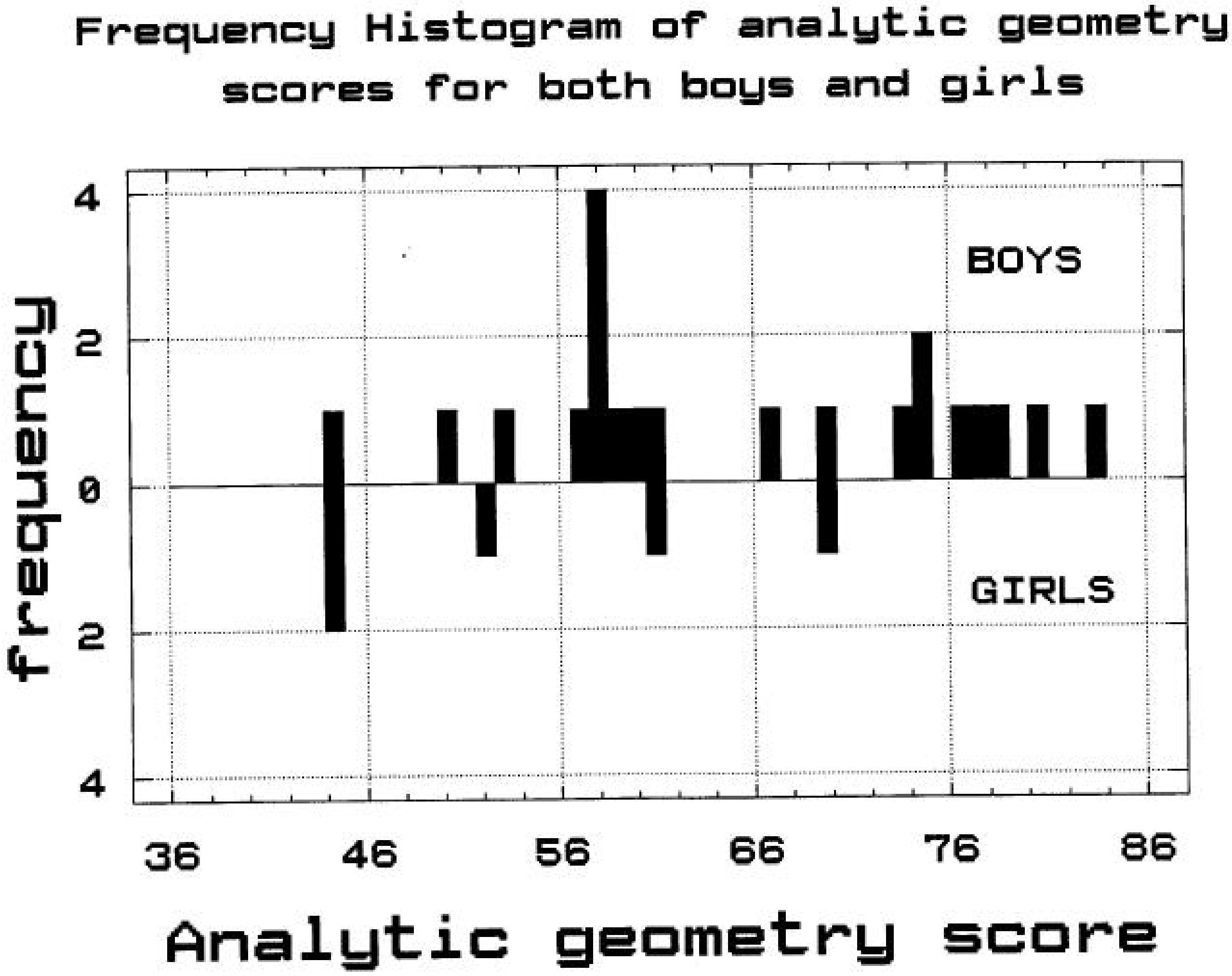


```
                         Two-Sample Analysis Results
------------------------------------------------------------------------------
                                       ANALB        ANALG        Pooled
mple Statistics: Number of Obs.        22           5            27
                 Average               64.1818      54.2         62.3333
                 Variance              162.346      127.2        156.723
                 Std. Deviation        12.7415      11.2783      12.5189
                 Median                60.5         52           60

fference between Means = 9.98182
nf. Interval For Diff. in Means:       95    Percent
Equal Vars.)    Sample 1 - Sample 2    -2.79506 22.7587     25 D.F.
Unequal Vars.)  Sample 1 - Sample 2    -3.75874 23.7224    6.6 D.F.

tio of Variances = 1.27631
nf. Interval for Ratio of Variances:   0     Percent
                Sample 1 - Sample 2

pothesis Test for H0: Diff = 0         Computed t statistic = 1.60938
                      vs Alt: NE       Sig. Level = 0.12009
                  at Alpha = 0.05      so do not reject H0.
```



SUMMARY OF CHI^2 AND KOLMOGOROV–SMIRNOV TESTS FOR "GOODNESS OF FIT" TO THE NORMAL DISTRIBUTION FOR VARIOUS TEST SCORES OF BOTH BOYS AND GIRLS

BOYS	# of cases	chi^2 statistic	K–S statistic	GIRLS	# of cases	chi^2 statistic	K–S statistic
OTIS	152	.312	.999	OTIS	106	.257	.54
PSATV	72	0	.19	PSATV	63	.12	.207
PSATM	72	.72	.54	PSATM	63	0	.35
SATV	76	.176	.188	SATV	47	0	.393
SATM	76	0	.126	SATM	47	.059	.155
GEOM	123	.274	.999	GEOM	77	.315	.199
EALG	142	.534	.999	EALG	99	.017	.265
TRIG	95	.126	.24	TRIG	53	.012	.999
IALG	50	.669	.199	IALG	25	.142	.999
ANAL	22	.032	.199	ANAL	5	N/A	1

BOYS AND GIRLS	# of cases	chi^2 statistic	K–S statistic
OTIS	258	.069	.369
PSATV	135	0	.093
PSATM	135	0	.179
SATV	123	0	.05
SATM	123	0	.026
GEOM	200	.182	.358
EALG	241	.1	.418
TRIG	148	0	.125
IALG	75	.045	1
ANAL	27	.127	1

TABLE 1

SUMMARY OF SIGNIFICANCE LEVELS OF BOYS VS GIRLS WITH THE t–TEST USING INDEPENDENT MEANS

BOYS vs GIRLS	t_{ind} significance levels
OTIS	.434
PSATV	.223
PSATM	.151
SATV	.713
SATM	.258
GEOM	.71
EALG	.459
TRIG	.462
IALG	.141
ANAL	.12

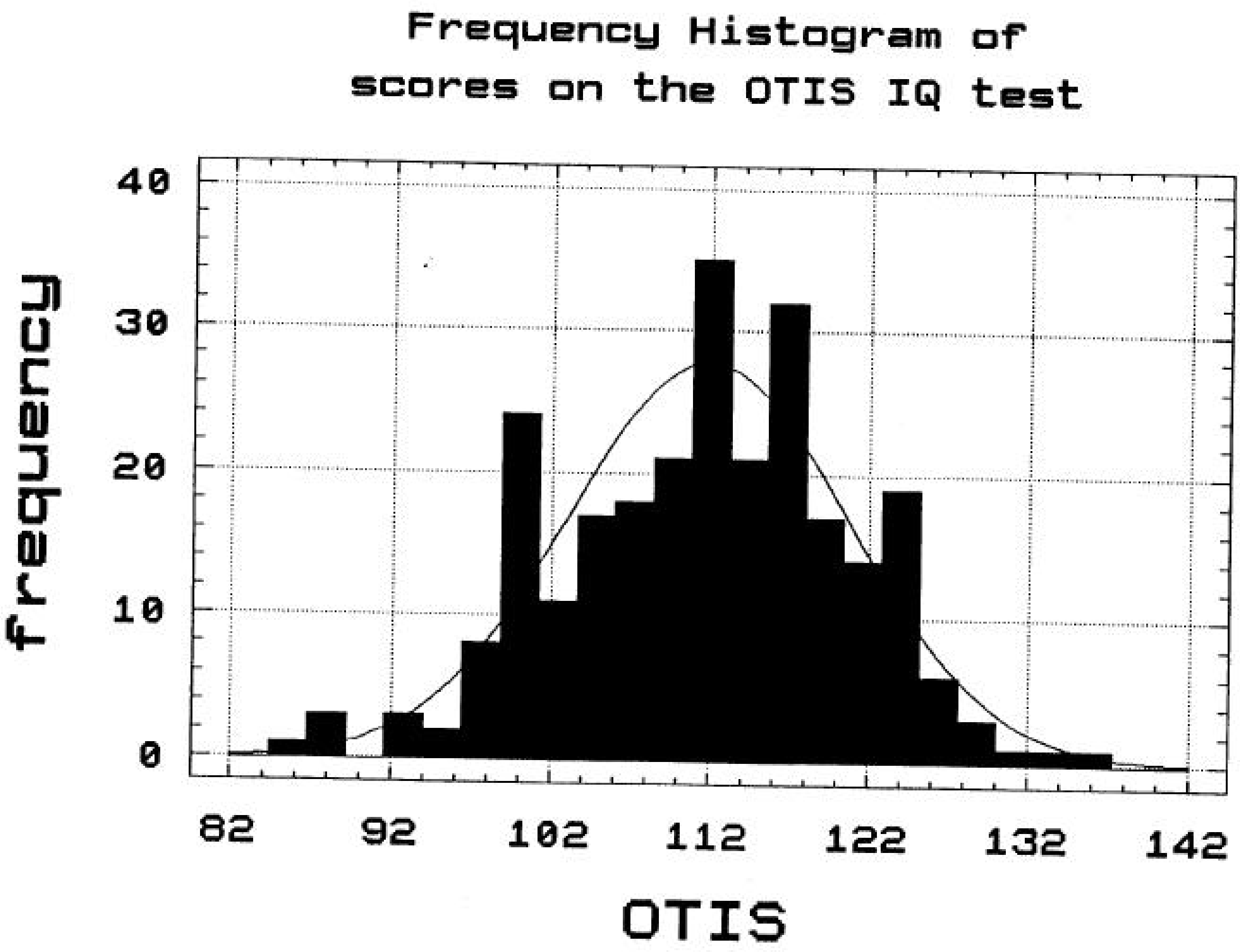
TABLE 2

SUMMARY OF 2–VARIABLE PRODUCT MOMENT CORRELATION COEFFICIENTS OF TEST SCORES FOR HIGH SCHOOL STUDENTS

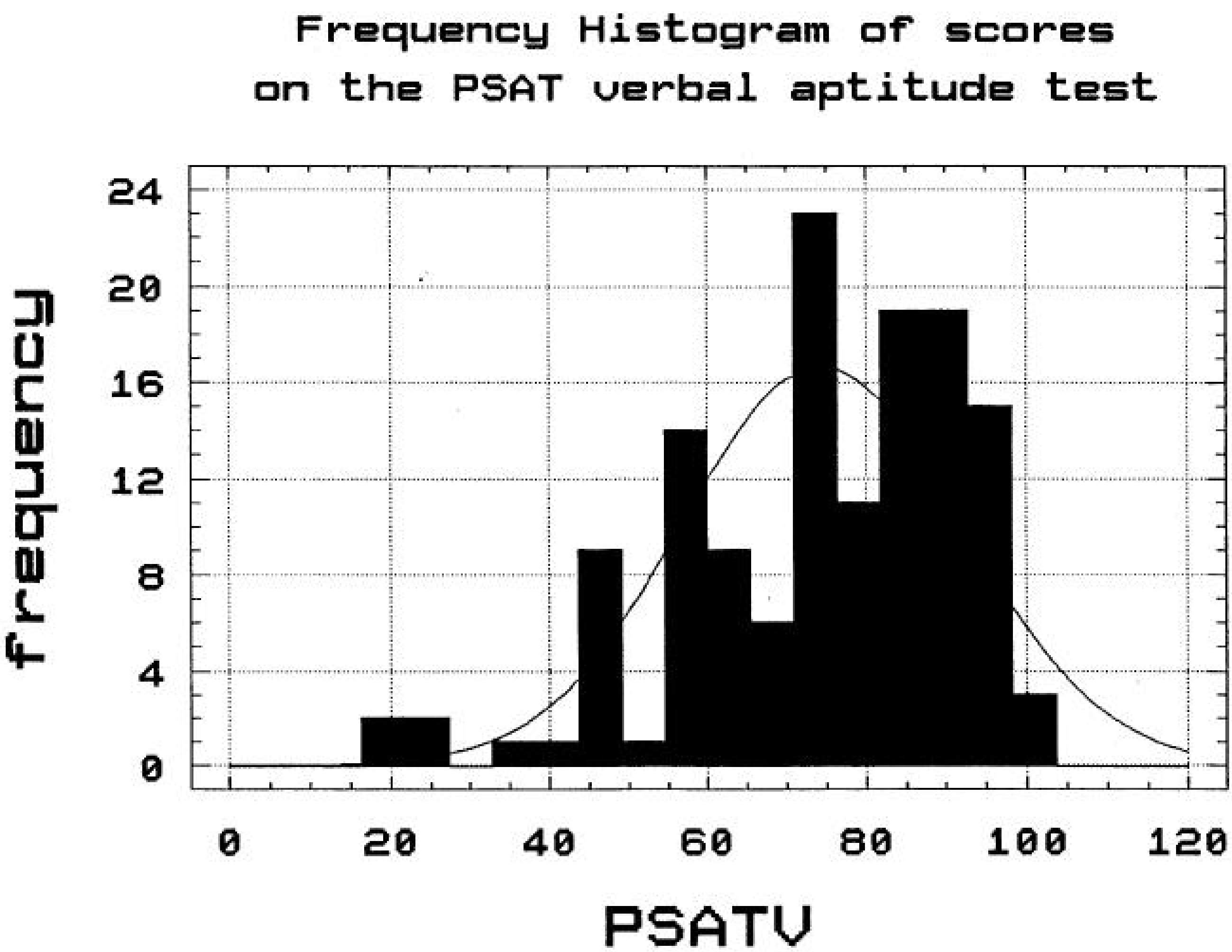
X/Y	PSATV	PSATM	SATV	SATM	GEOM	EALG	TRIG	IALG	ANAL
OTIS	.656 N=132	.568 N=132	.7 N=119	.644 N=119	.473 N=197	.513 N=236	.291 N=146	.414 N=74	.286 N=27
PSATV		.533 N=135	.604 N=76	.396 N=76	.335 N=102	.356 N=130	.266 N=78	.414 N=34	.114 N=12
PSATM			.218 N=76	.4 N=76	.439 N=102	.643 N=131	.294 N=78	.485 N=34	–.197 N=12
SATV				.578 N=123	.226 N=104	.416 N=111	.358 N=83	.241 N=58	.238 N=21
SATM					.284 N=104	.612 N=111	.387 N=83	.488 N=58	.164 N=21
GEOM						.549 N=191	.433 N-104	.363 N=63	.286 N=25
EALG							.562 N=136	.497 N=62	.201 N=23
TRIG								.565 N=63	.436 N=20
IALG									.545 N=24

TABLE 3

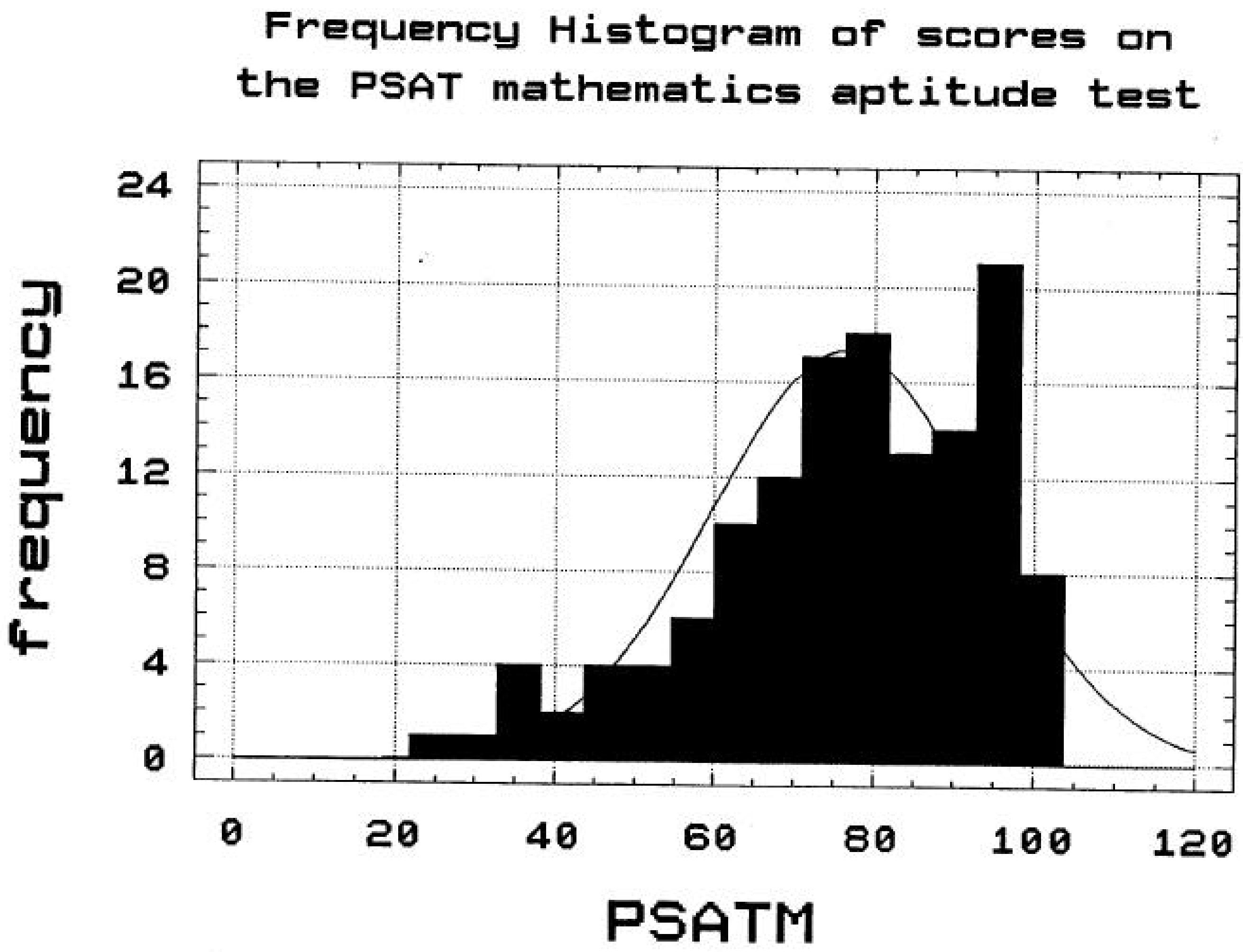
Variable:	OTIS
Sample size	258
Average	111.837
Median	112
Mode	115
Geometric mean	111.475
Variance	79.6388
Standard deviation	8.92406
Standard error	0.555587
Minimum	85
Maximum	135
Range	50
Lower quartile	106
Upper quartile	118
Interquartile range	12
Skewness	-0.252597
Standardized skewness	-1.65639
Kurtosis	-0.0972377
Standardized kurtosis	-0.318815



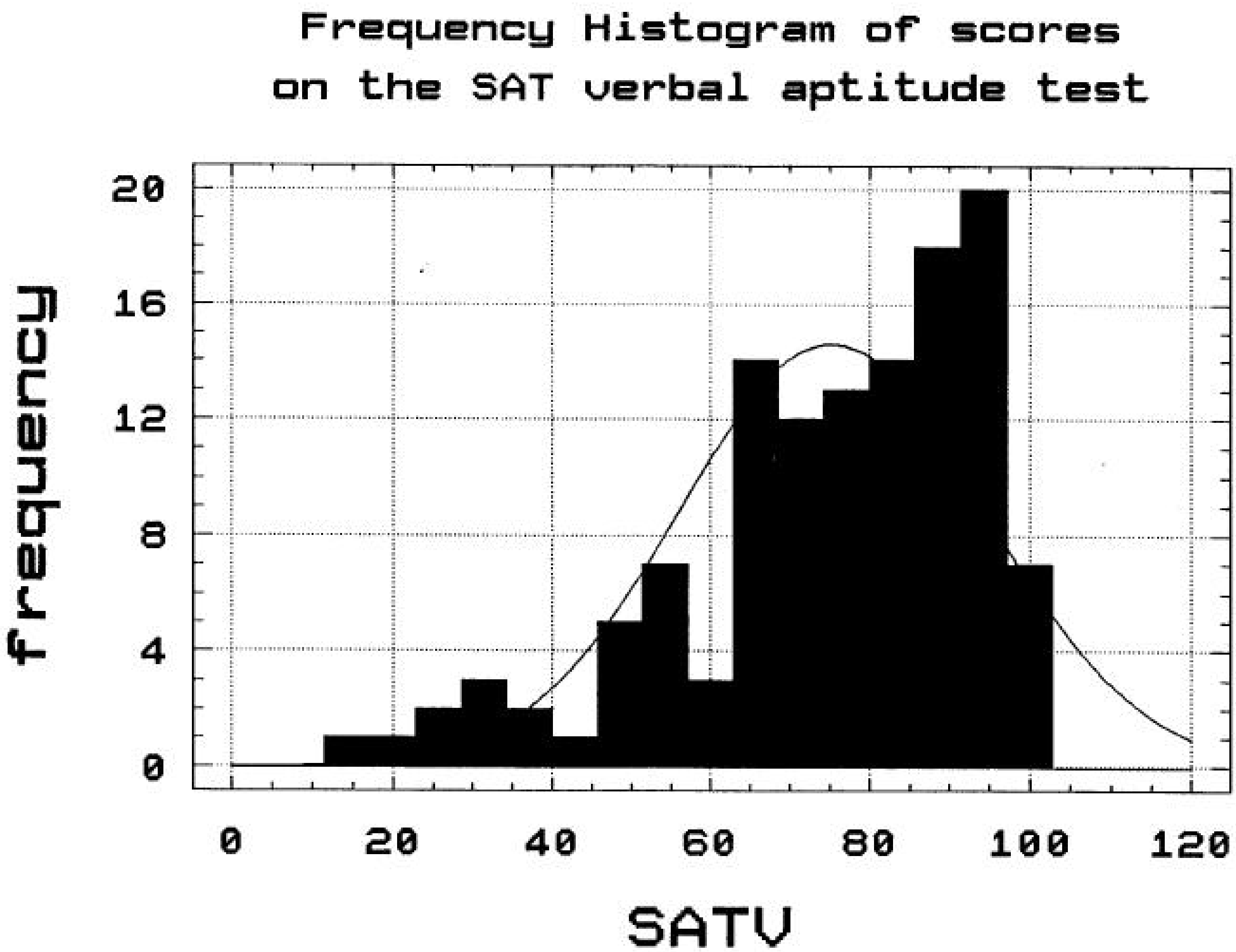
Variable:	PSATV
Sample size	135
Average	74.3481
Median	76
Mode	76
Geometric mean	71.6108
Variance	312.094
Standard deviation	17.6662
Standard error	1.52046
Minimum	21
Maximum	99
Range	78
Lower quartile	63
Upper quartile	89
Interquartile range	26
Skewness	-0.920881
Standardized skewness	-4.36812
Kurtosis	0.5559
Standardized kurtosis	1.31843



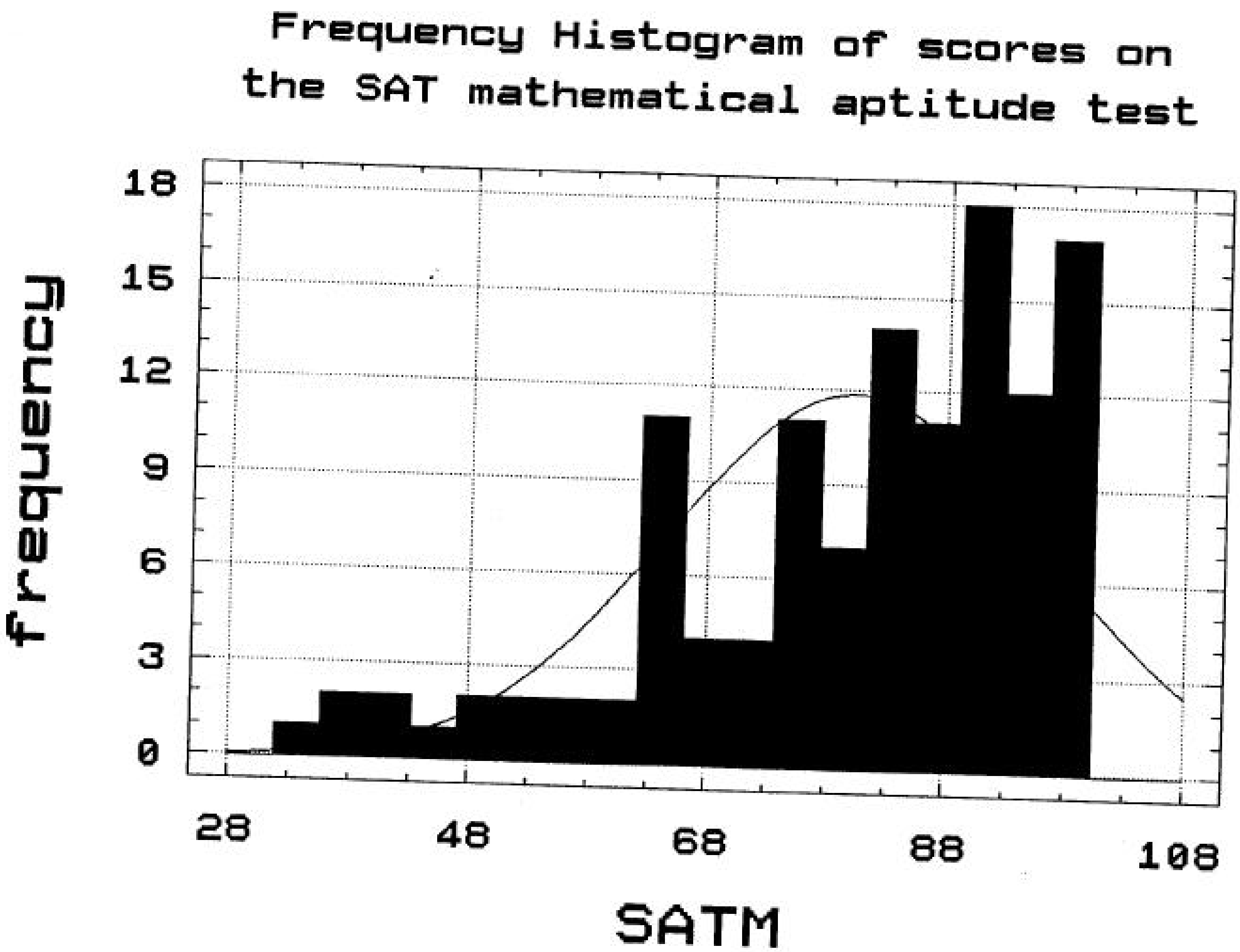
Variable:	PSATM
Sample size	135
Average	76.4963
Median	78
Mode	99
Geometric mean	74.2102
Variance	288.998
Standard deviation	16.9999
Standard error	1.46312
Minimum	26
Maximum	99
Range	73
Lower quartile	67
Upper quartile	91
Interquartile range	24
Skewness	-0.76921
Standardized skewness	-3.64868
Kurtosis	0.154011
Standardized kurtosis	0.365269



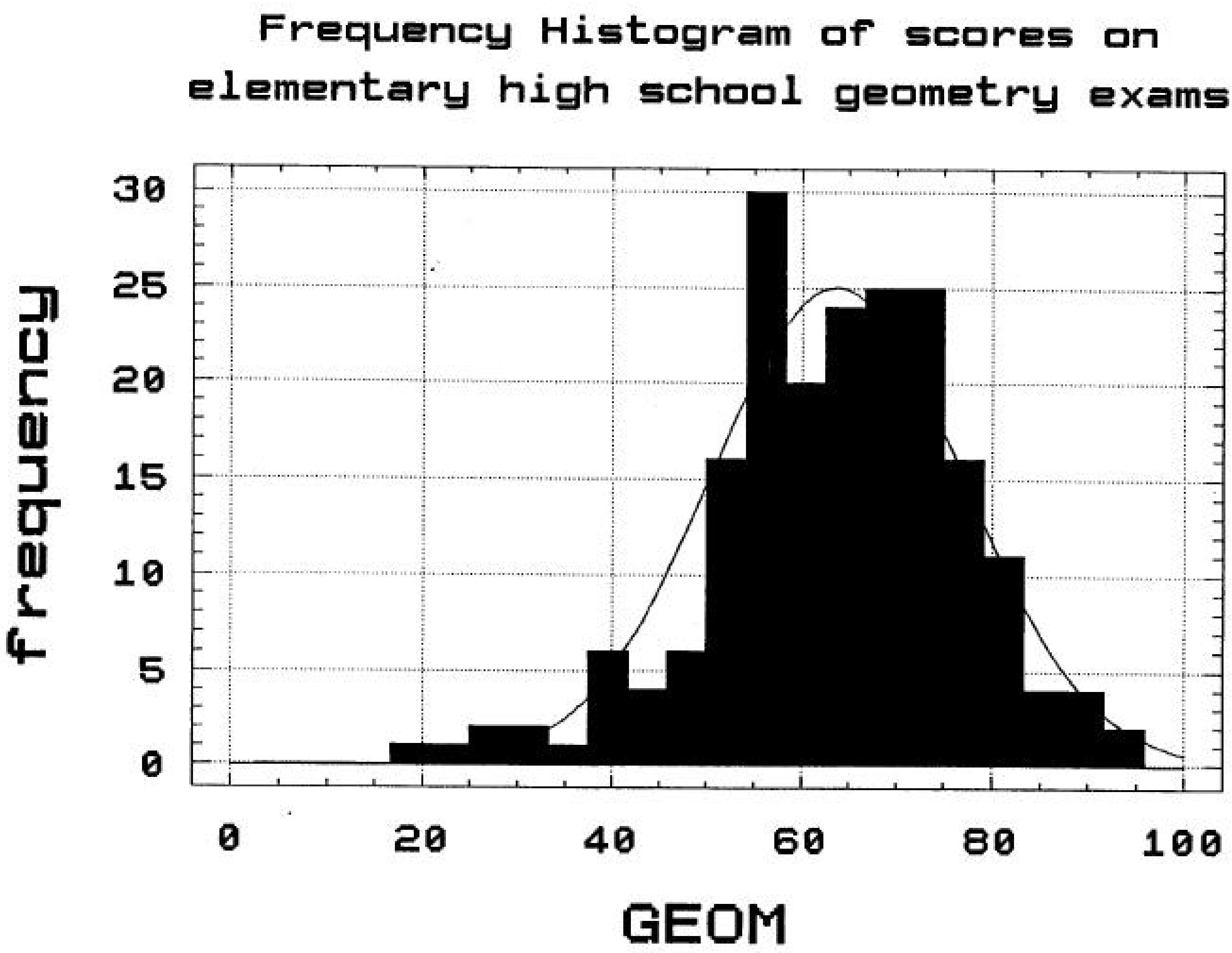
Variable:	SATV
Sample size	123
Average	75.2602
Median	80
Mode	94
Geometric mean	71.8977
Variance	369.112
Standard deviation	19.2123
Standard error	1.73231
Minimum	15
Maximum	99
Range	84
Lower quartile	65
Upper quartile	91
Interquartile range	26
Skewness	-1.02739
Standardized skewness	-4.65169
Kurtosis	0.571758
Standardized kurtosis	1.29437



Variable:	SATM
Sample size	123
Average	80.1626
Median	84
Mode	74
Geometric mean	78.2832
Variance	249.383
Standard deviation	15.7919
Standard error	1.42391
Minimum	32
Maximum	99
Range	67
Lower quartile	70
Upper quartile	92
Interquartile range	22
Skewness	-0.995292
Standardized skewness	-4.50637
Kurtosis	0.418914
Standardized kurtosis	0.948358

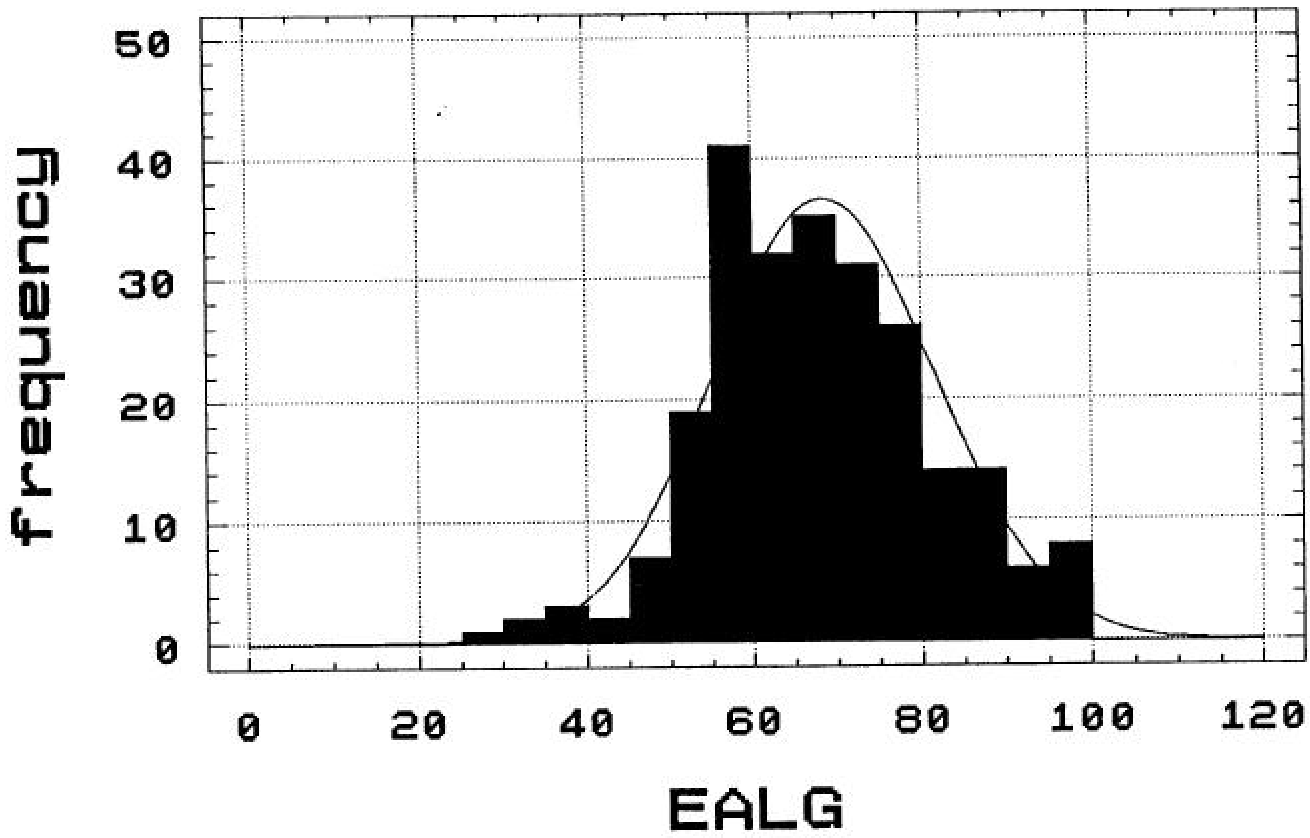


Variable:	GEOM
Sample size	200
Average	63.69
Median	64.5
Mode	67
Geometric mean	62.0531
Variance	176.446
Standard deviation	13.2833
Standard error	0.939271
Minimum	18
Maximum	95
Range	77
Lower quartile	56
Upper quartile	72.5
Interquartile range	16.5
Skewness	-0.509625
Standardized skewness	-2.94232
Kurtosis	0.821249
Standardized kurtosis	2.37074



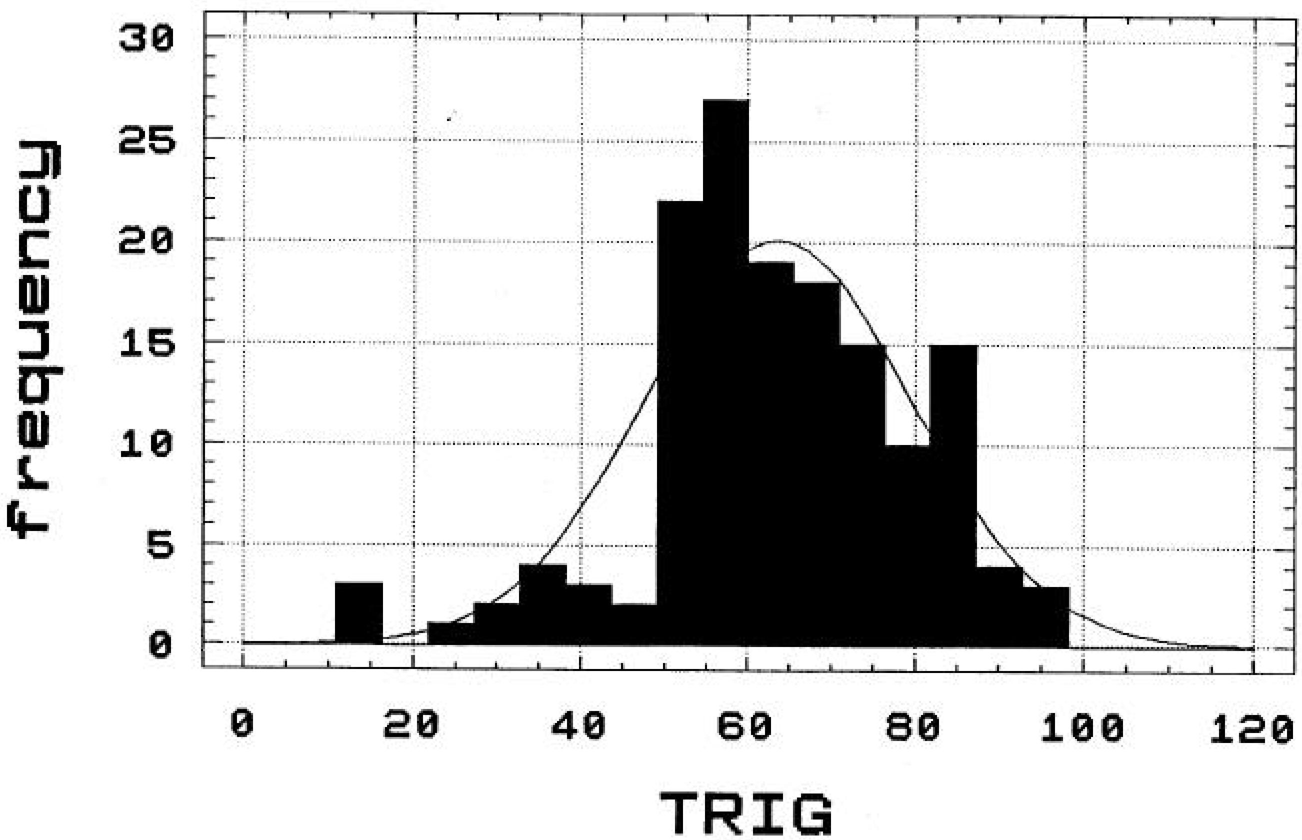
Variable:	EALG
Sample size	241
Average	68.3859
Median	68
Mode	60
Geometric mean	67.0486
Variance	173.963
Standard deviation	13.1895
Standard error	0.849611
Minimum	28
Maximum	97
Range	69
Lower quartile	60
Upper quartile	77
Interquartile range	17
Skewness	0.023638
Standardized skewness	0.149811
Kurtosis	0.0838099
Standardized kurtosis	0.265582

Frequency Histogram of scores on elementary high school algebra exams



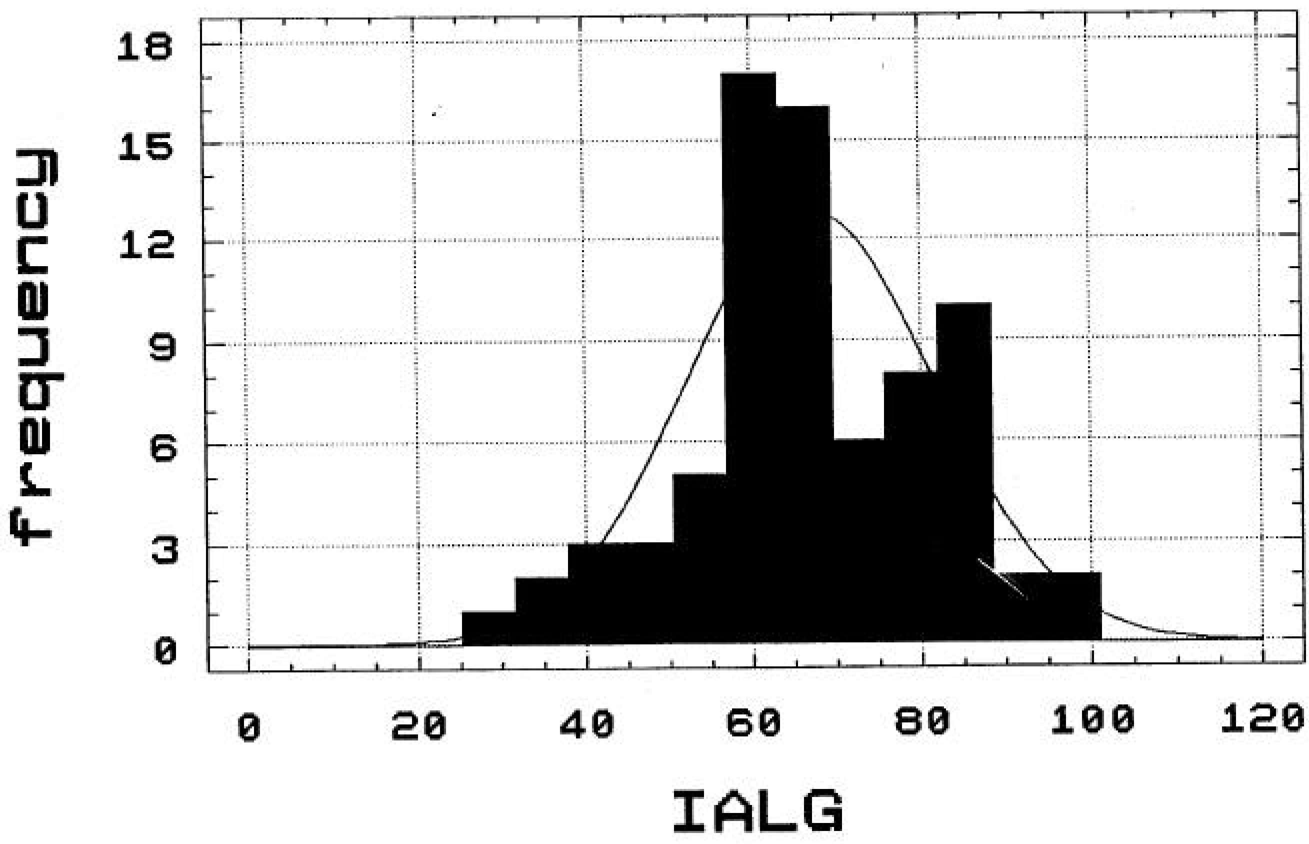
Variable:	TRIG
Sample size	148
Average	63.6014
Median	63
Mode	74
Geometric mean	60.9773
Variance	257.221
Standard deviation	16.0381
Standard error	1.31832
Minimum	12
Maximum	98
Range	86
Lower quartile	54.5
Upper quartile	74
Interquartile range	19.5
Skewness	-0.516299
Standardized skewness	-2.56423
Kurtosis	0.797877
Standardized kurtosis	1.98135

Frequency Histogram of scores on high school elementary trigonometry exams

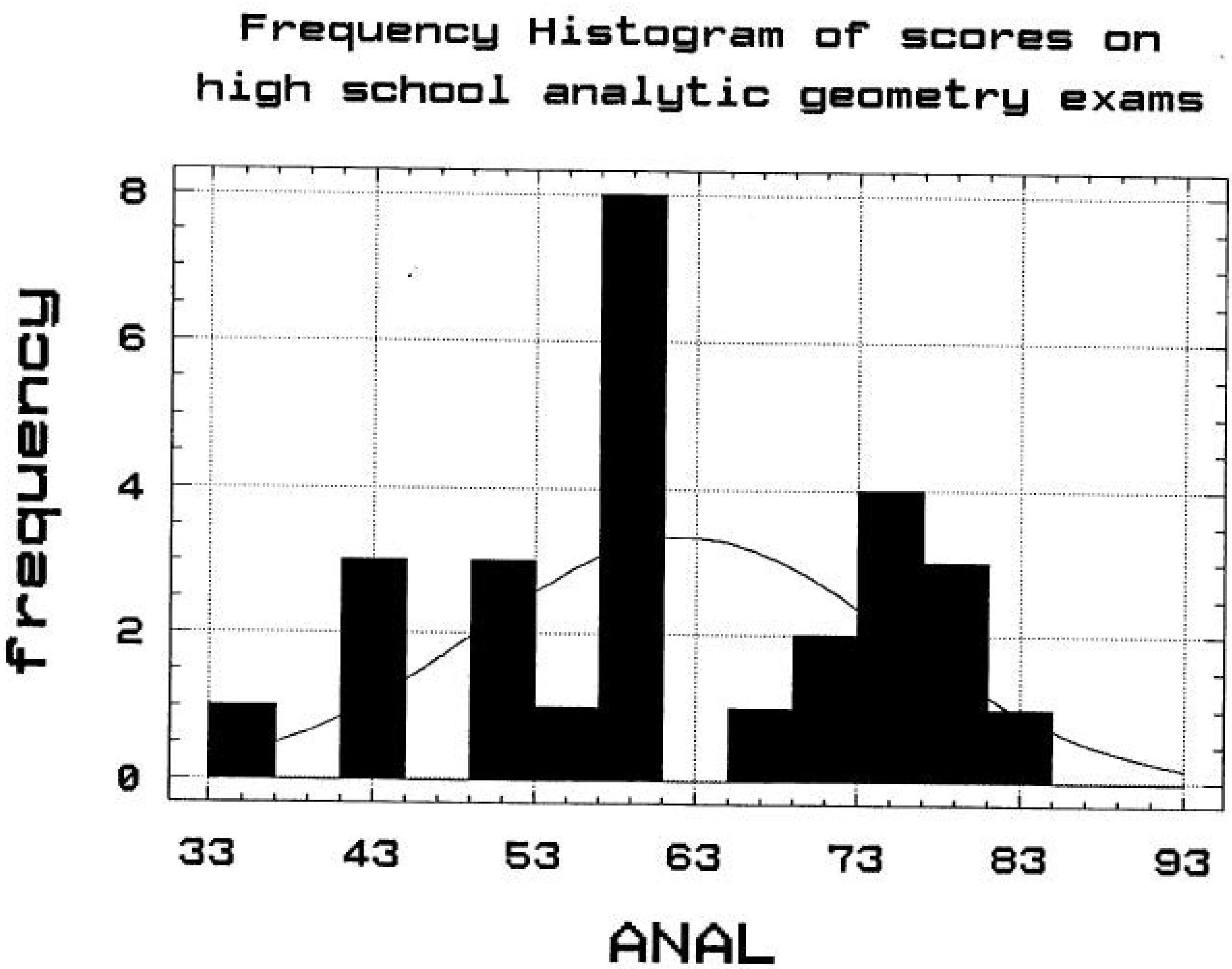


Variable:	IALG
Sample size	75
Average	66.8667
Median	66
Mode	66
Geometric mean	65.0594
Variance	216.279
Standard deviation	14.7064
Standard error	1.69815
Minimum	27
Maximum	98
Range	71
Lower quartile	60
Upper quartile	78
Interquartile range	18
Skewness	-0.307855
Standardized skewness	-1.08843
Kurtosis	0.191058
Standardized kurtosis	0.337746

Frequency Histogram of scores on high school intermediate algebra exams



Variable:	ANAL
Sample size	27
Average	62.3333
Median	60
Mode	58
geometric mean	60.9784
Variance	166.308
Standard deviation	12.896
Standard error	2.48184
Minimum	36
Maximum	84
Range	48
Lower quartile	53
Upper quartile	75
Interquartile range	22
Skewness	-0.118338
Standardized skewness	-0.251032
Kurtosis	-0.809332
Standardized kurtosis	-0.858426



Regression Analysis - Linear model: Y = a+bX

Dependent variable: OTIS Independent variable: PSATV

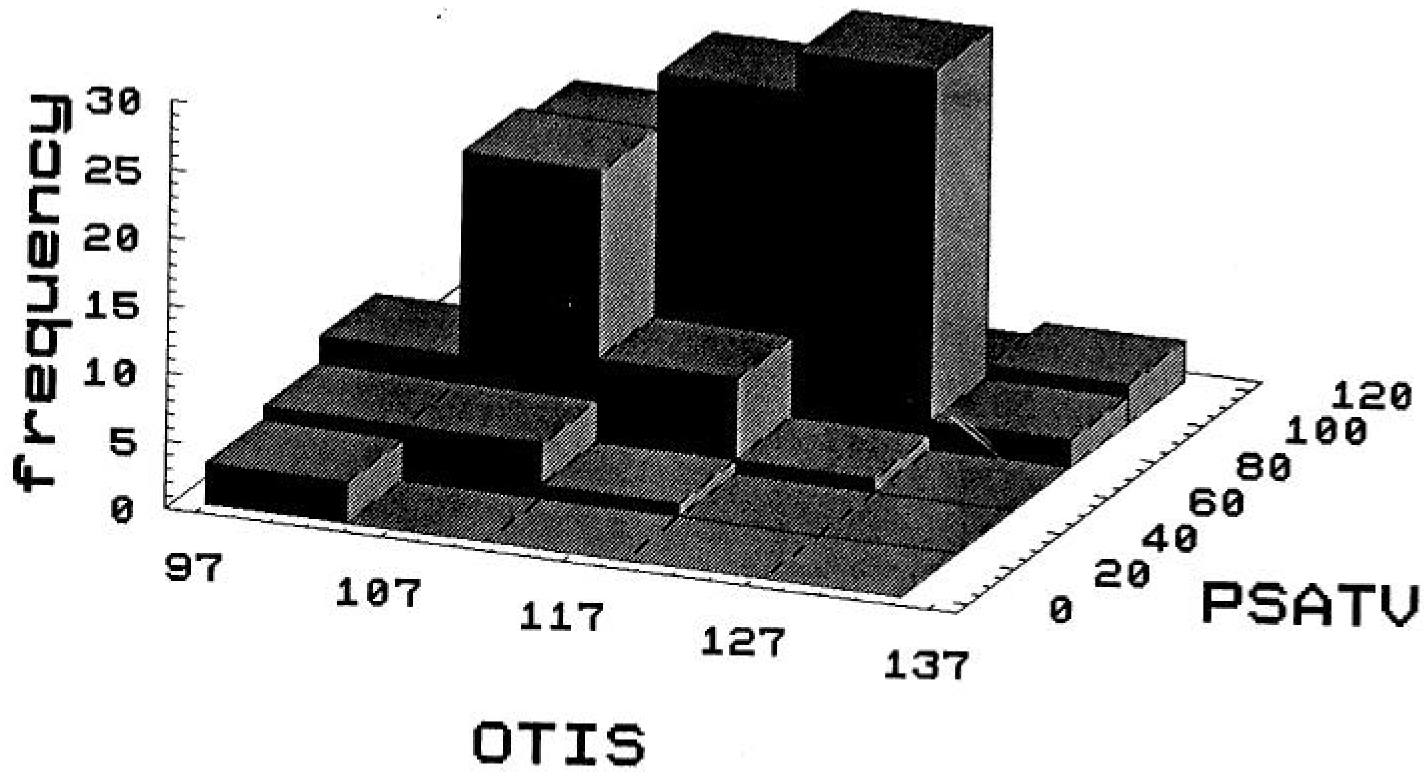
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	91.3654	2.33398	39.1457	.00000
Slope	0.302337	0.0305182	9.90676	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	3712.8372	1	3712.8372	98.144	.00000
Error	4917.9734	130	37.8306		
Total (Corr.)	8630.8106	131			

Correlation Coefficient = 0.655884 R-squared = 43.02 percent
Stnd. Error of Est. = 6.15066

Three-D Histogram of OTIS IQ scores vs PSAT verbal aptitude scores



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Regression Analysis - Linear model: Y = a+bX

Dependent variable: PSATV   Independent variable: OTI

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	-87.5839	16.396	-5.34178	.00000
Slope	1.42286	0.143626	9.90676	.00000

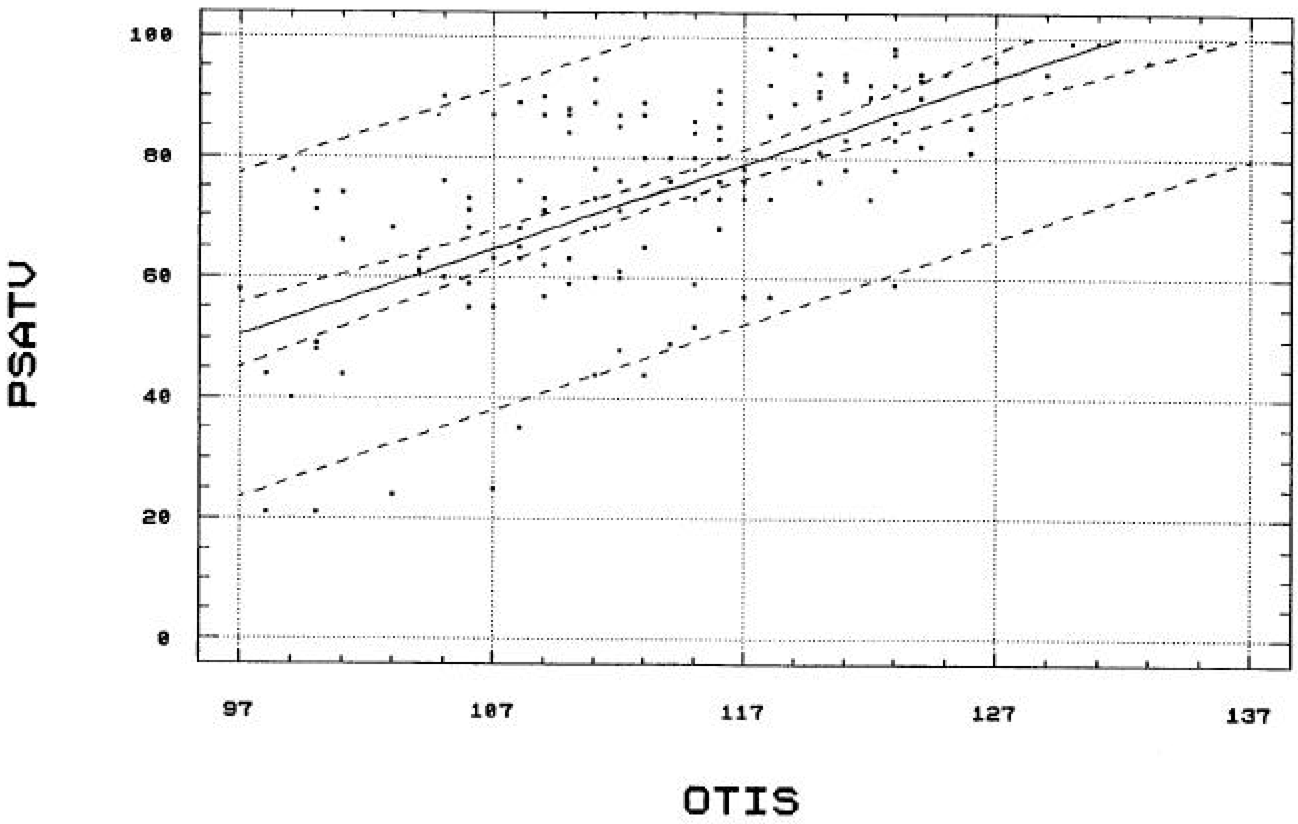
Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	17473.438	1	17473.438	98.14	.00000
Error	23145.077	130	178.039		
Total (Corr.)	40618.515	131			

Correlation Coefficient = 0.655884   R-squared = 43.02 percent

Stnd. Error of Est. = 13.3431

Regression of PSATV scores in verbal aptitude on OTIS IQ scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: PSATM — Independent variable: OTI[illegible]

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	-58.9236	17.283	-3.40934	.00087
Slope	1.19093	0.151395	7.86638	.00000

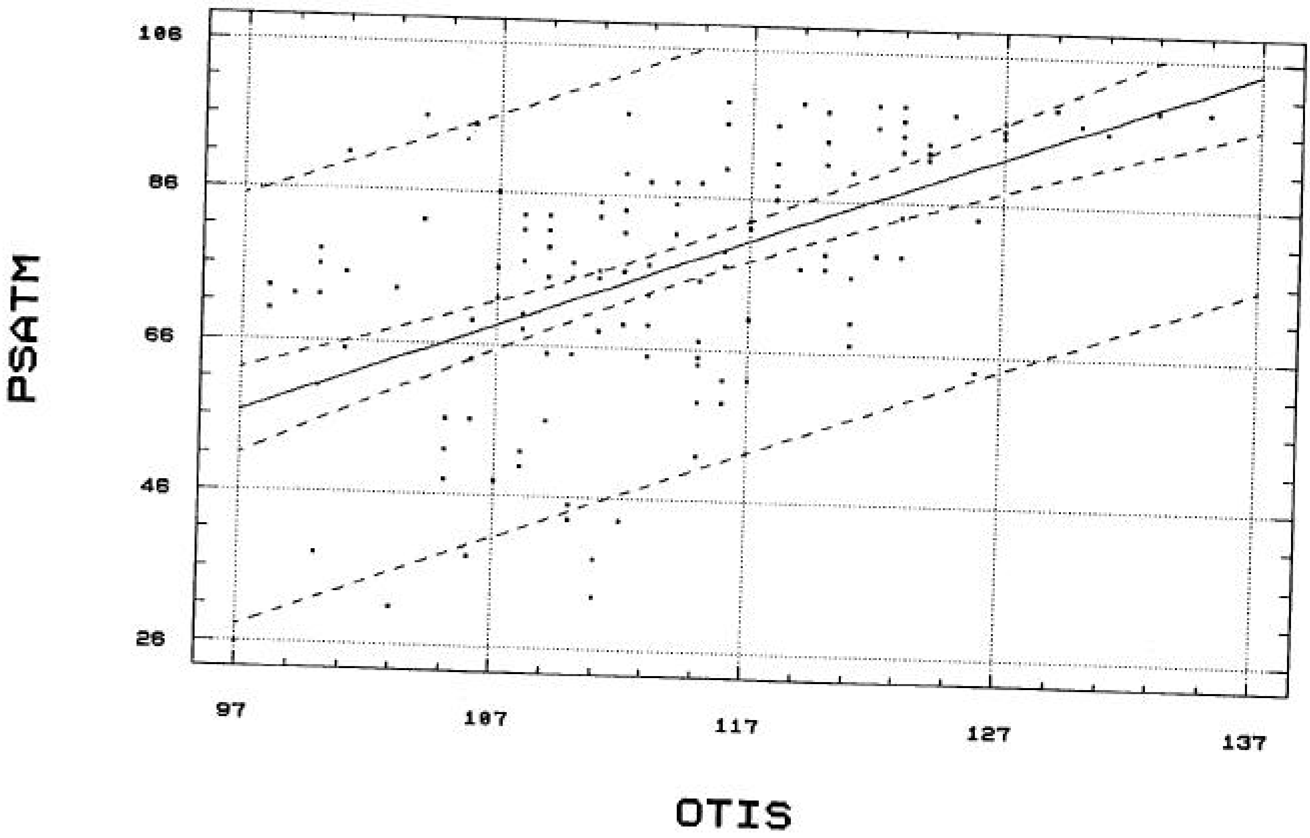
Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	12241.269	1	12241.269	61.88	.00000
Error	25716.996	130	197.823		
Total (Corr.)	37958.265	131			

Correlation Coefficient = 0.567885
Stnd. Error of Est. = 14.065

R-squared = 32.25 percent

Regression of PSATM scores in mathematics on OTIS IQ scores



Regression Analysis - Linear model: Y = a+bX

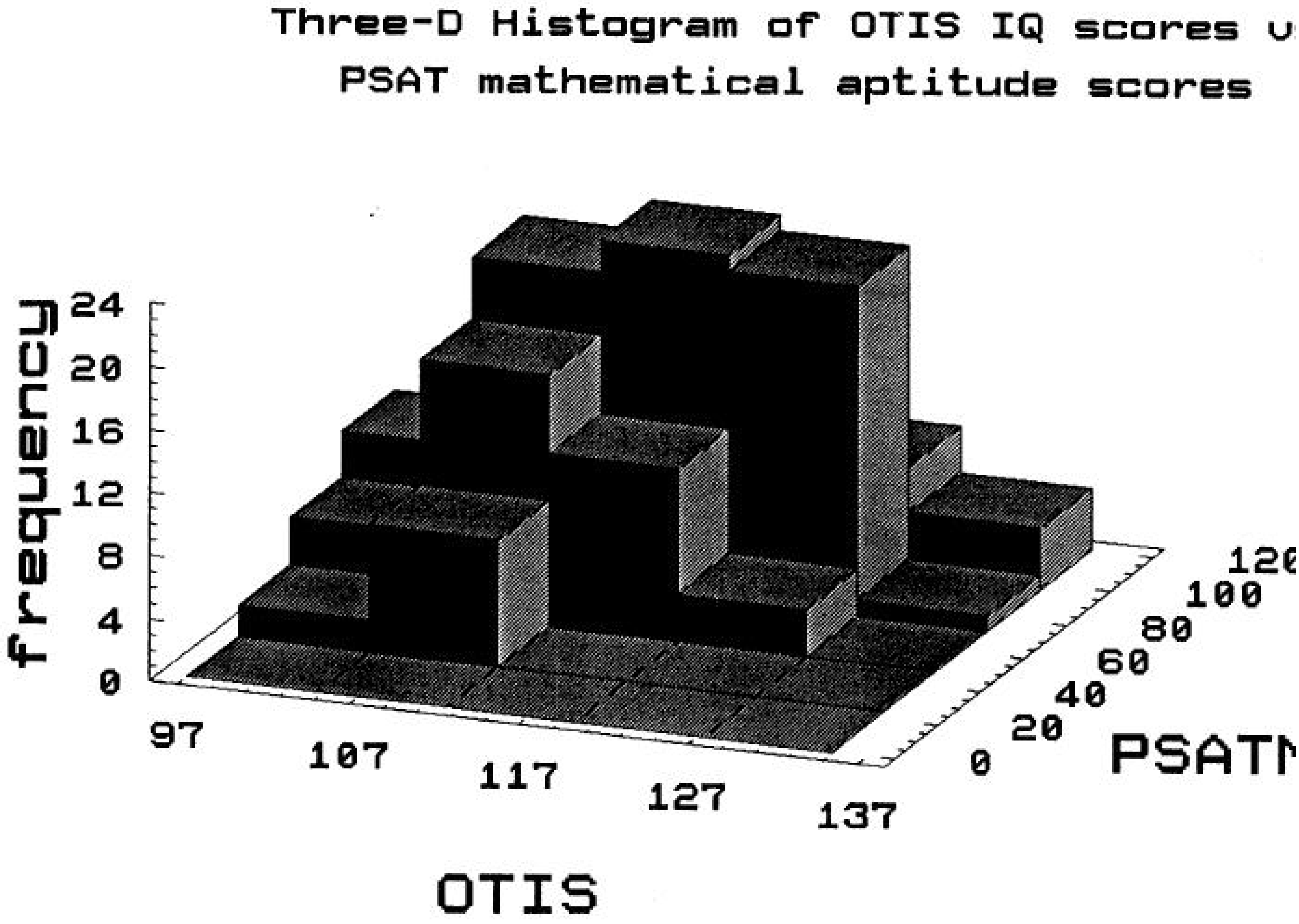
Dependent variable: OTIS   Independent variable: PSAT

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	93.1045	2.7037	34.4359	.00000
Slope	0.27079	0.0344237	7.86638	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	2783.3748	1	2783.3748	61.880	.00000
Error	5847.4358	130	44.9803		
Total (Corr.)	8630.8106	131			

Correlation Coefficient = 0.567885   R-squared = 32.25 percent
Stnd. Error of Est. = 6.70673



Regression Analysis - Linear model: Y = a+bX

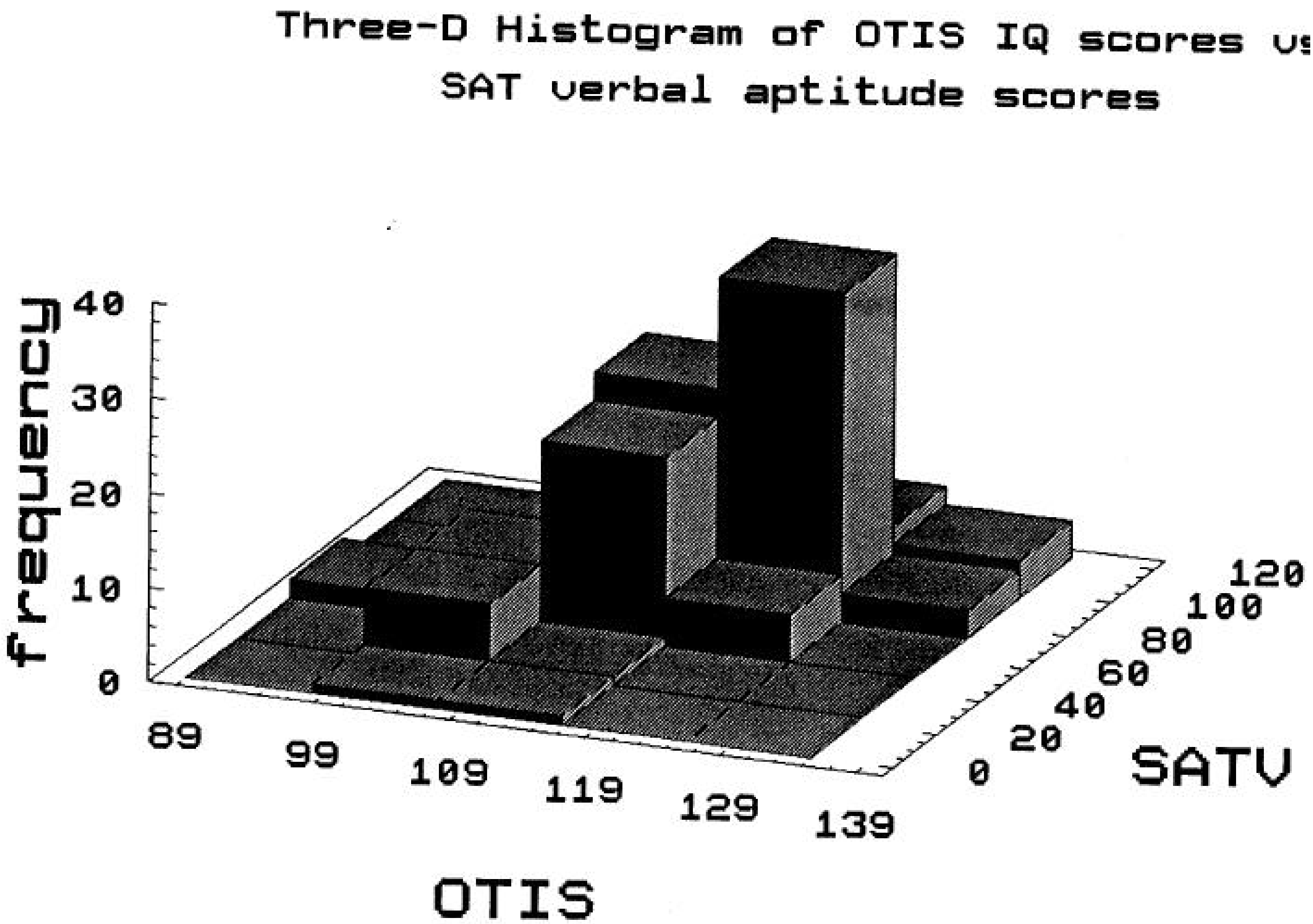
Dependent variable: OTIS Independent variable: SAT'

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	92.3694	2.18752	42.2257	.00000
Slope	0.298898	0.0281639	10.6128	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	3959.5067	1	3959.5067	112.632	.00000
Error	4113.0647	117	35.1544		
Total (Corr.)	8072.5714	118			

Correlation Coefficient = 0.700349 R-squared = 49.05 percent
Stnd. Error of Est. = 5.92911



Regression Analysis - Linear model: Y = a+bX

Dependent variable: SATV Independent variable: OTI

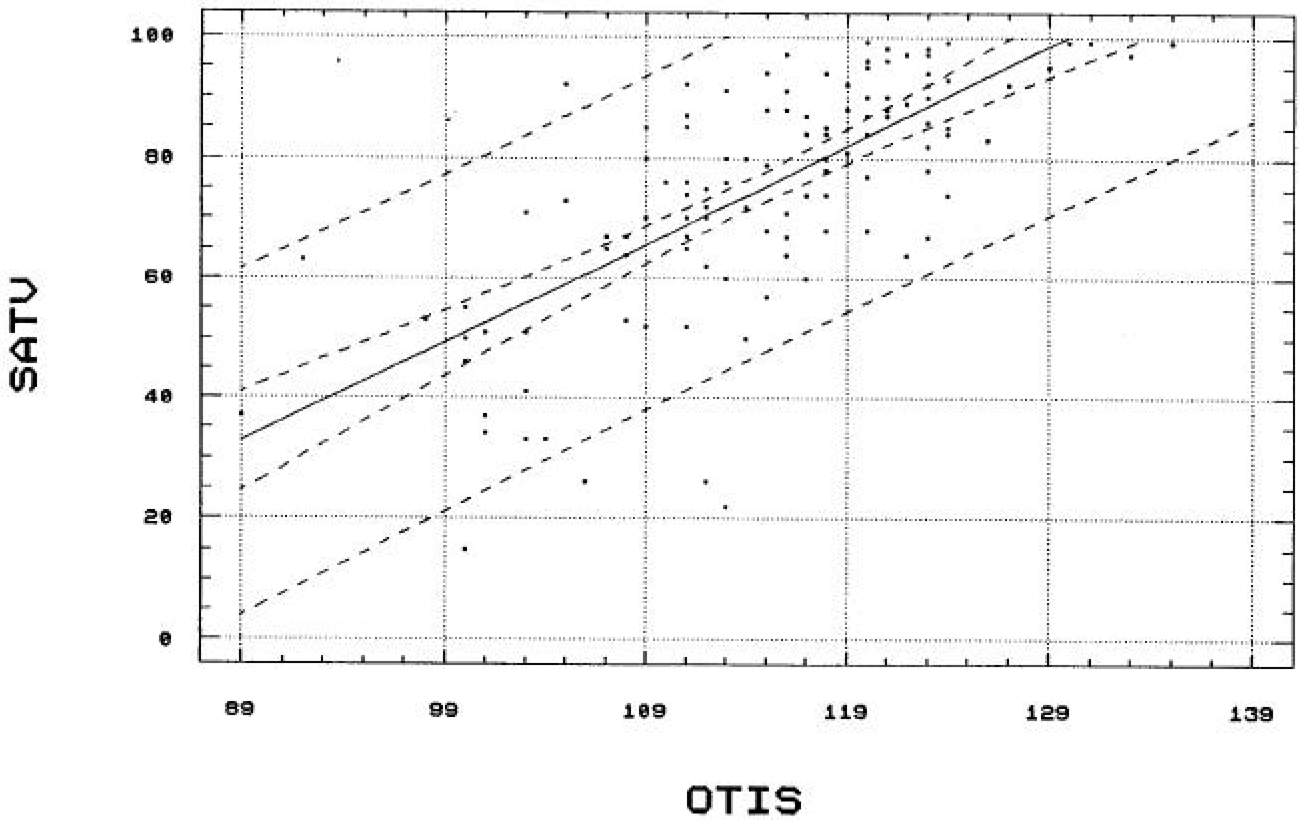
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	-113.244	17.8052	-6.36017	.00000
Slope	1.64099	0.154623	10.6128	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	21738.180	1	21738.180	112.63	.00000
Error	22581.232	117	193.002		
Total (Corr.)	44319.412	118			

Correlation Coefficient = 0.700349 R-squared = 49.05 percent
Stnd. Error of Est. = 13.8925

Regression of SATV scores in verbal aptitude on OTIS IQ scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: SATM   Independent variable: OT

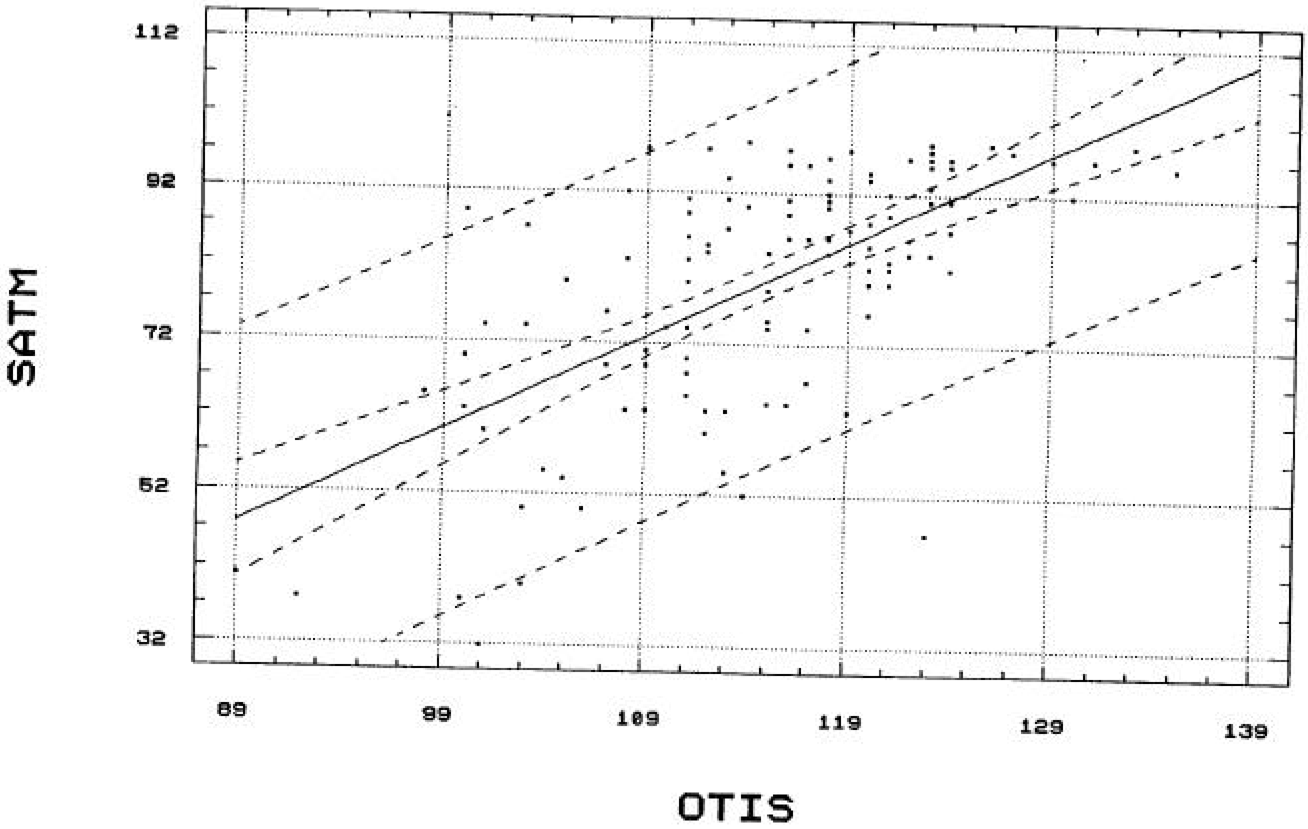
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	-62.8019	15.7462	-3.98838	.00012
Slope	1.24418	0.136743	9.09868	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	12496.166	1	12496.166	82.79	.00000
Error	17660.623	117	150.945		
Total (Corr.)	30156.790	118			

Correlation Coefficient = 0.643718   R-squared = 41.44 percent
Stnd. Error of Est. = 12.286

Regression of SATM scores in mathematics on OTIS IQ scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: OTIS — Independent variable: SATM

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	88.1796	2.98937	29.4977	.00000
Slope	0.33305	0.0366042	9.09868	.00000

Analysis of Variance

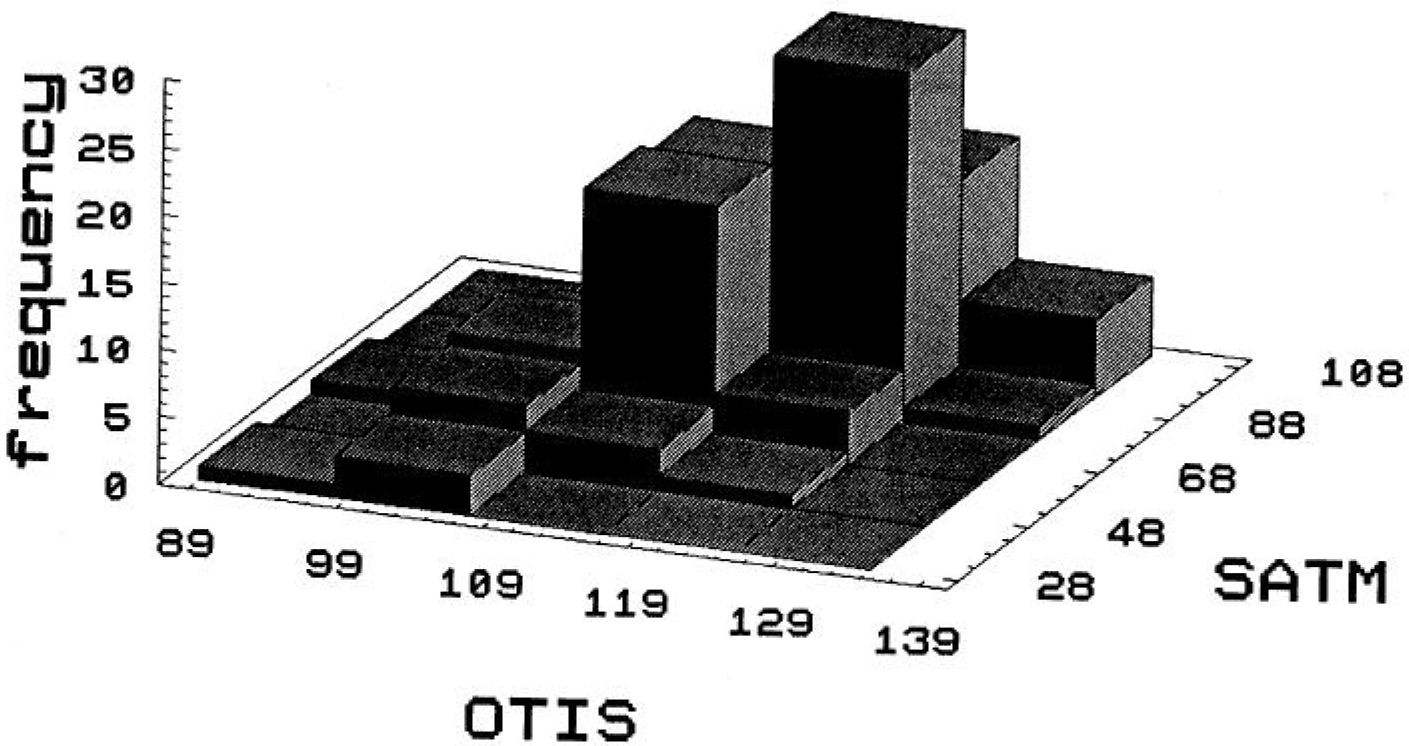
Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	3345.0575	1	3345.0575	82.786	.00000
Error	4727.5139	117	40.4061		
Total (Corr.)	8072.5714	118			

Correlation Coefficient = 0.643718

R-squared = 41.44 percent

Stnd. Error of Est. = 6.35658

Three-D Histogram of OTIS IQ scores vs SAT mathematical aptitude scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: OTIS

Independent variable: GEOM

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	91.09	2.76418	32.9537	.00000
Slope	0.318062	0.0423952	7.50232	.00000

Analysis of Variance

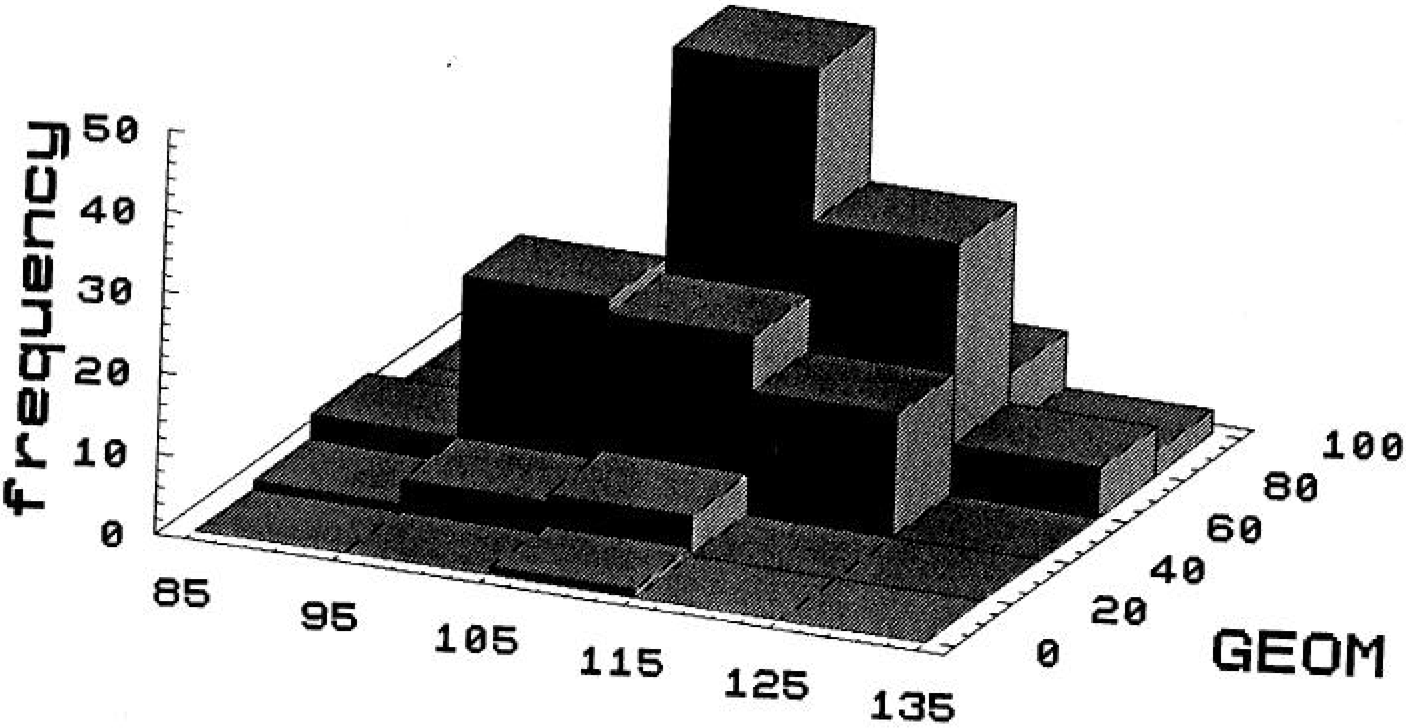
Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	3491.7742	1	3491.7742	56.285	.00000
Error	12097.343	195	62.038		
Total (Corr.)	15589.117	196			

Correlation Coefficient = 0.473274

Stnd. Error of Est. = 7.8764

R-squared = 22.40 percent

Three-D Histogram of OTIS IQ scores vs
Elementary high school geometry scores



Regression Analysis - Linear model: Y = a+bX

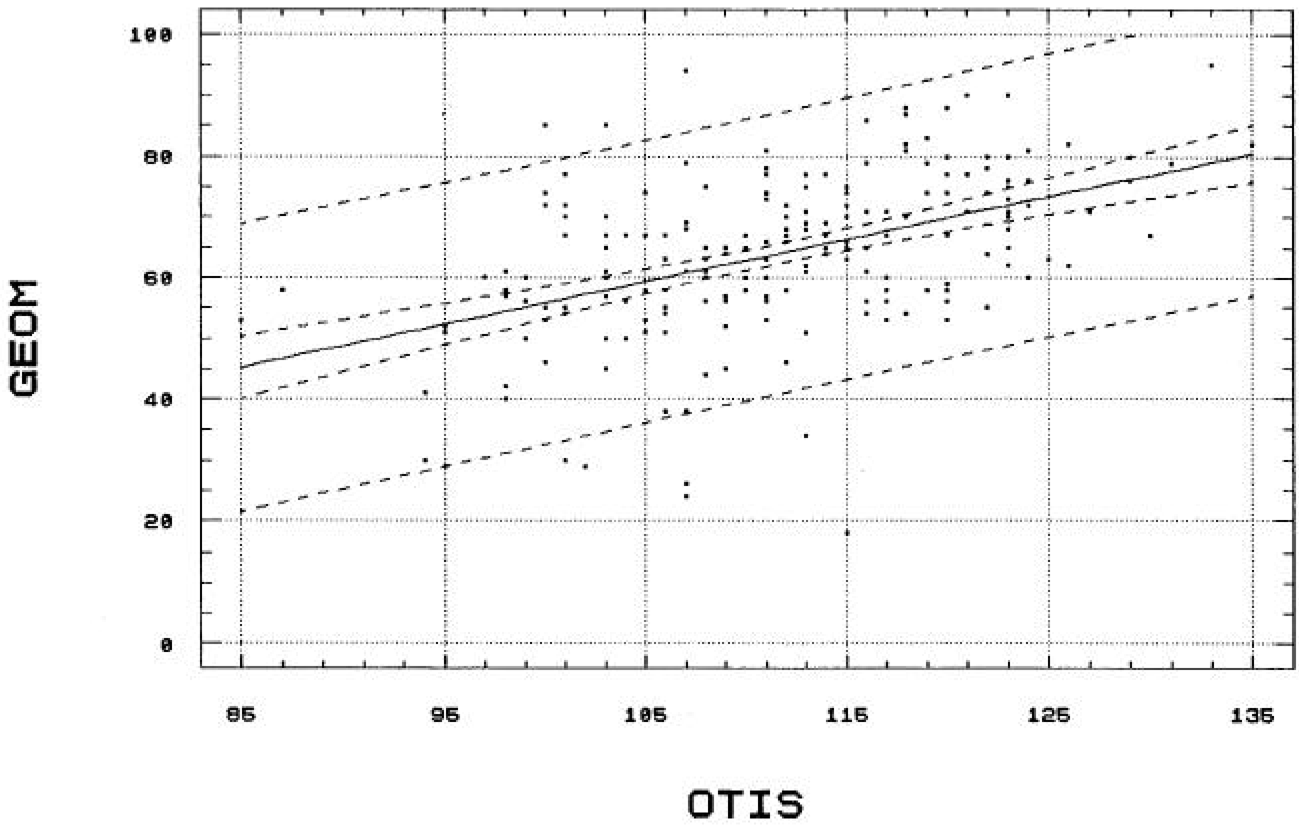
Dependent variable: GEOM				Independent variable: OTI
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	-14.6054	10.4898	-1.39234	.16540
Slope	0.704227	0.0938679	7.50232	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	7731.1951	1	7731.1951	56.285	.00000
Error	26784.927	195	137.359		
Total (Corr.)	34516.122	196			

Correlation Coefficient = 0.473274 R-squared = 22.40 percent
Stnd. Error of Est. = 11.72

Regression of high school GEOMetry scores on OTIS IQ scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: EALG   Independent variable: OTI

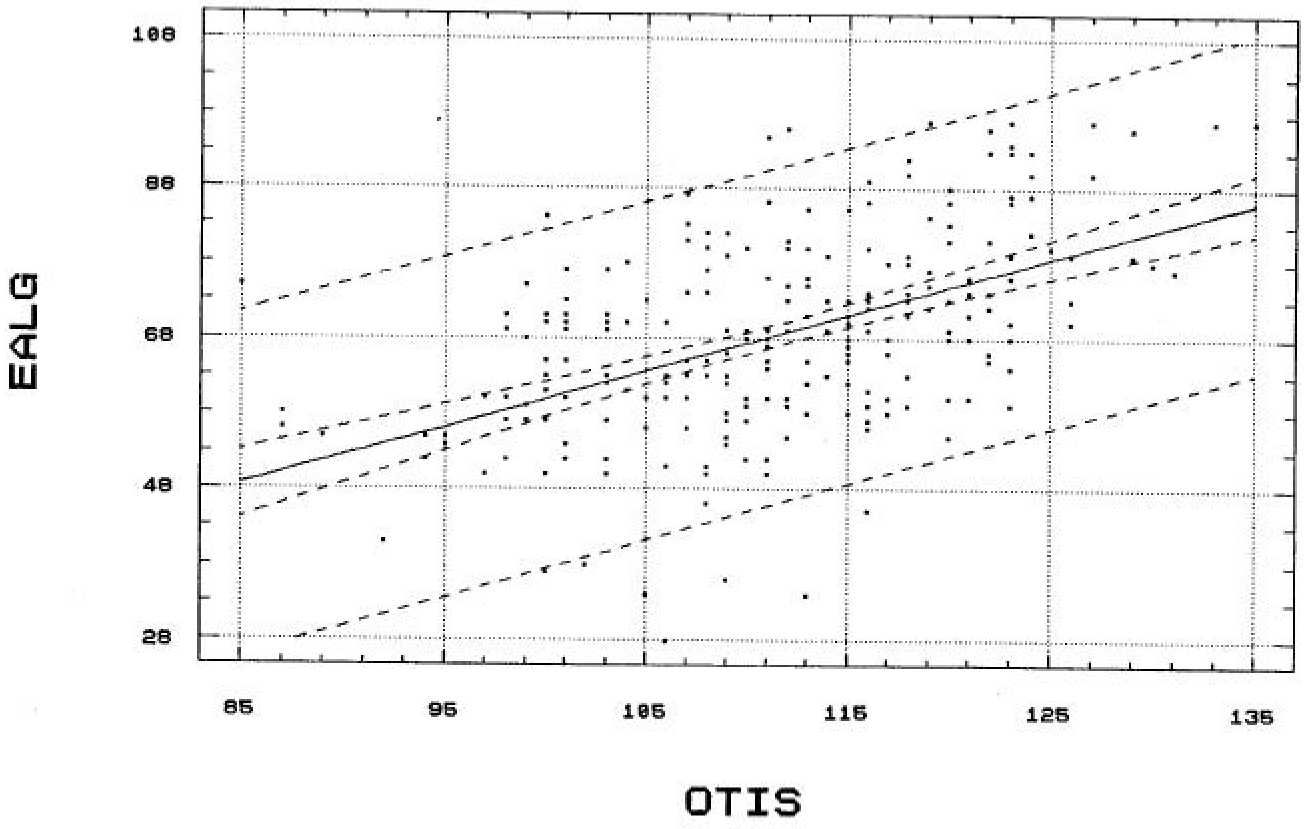
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	-14.9075	9.15554	-1.62825	.10482
Slope	0.747709	0.0817834	9.14255	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	10733.729	1	10733.729	83.59	.00000
Error	30049.119	234	128.415		
Total (Corr.)	40782.847	235			

Correlation Coefficient = 0.513023   R-squared = 26.32 percent
Stnd. Error of Est. = 11.332

Regression of elementary high school algebra on OTIS IQ scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: OTIS Independent variable: EAI

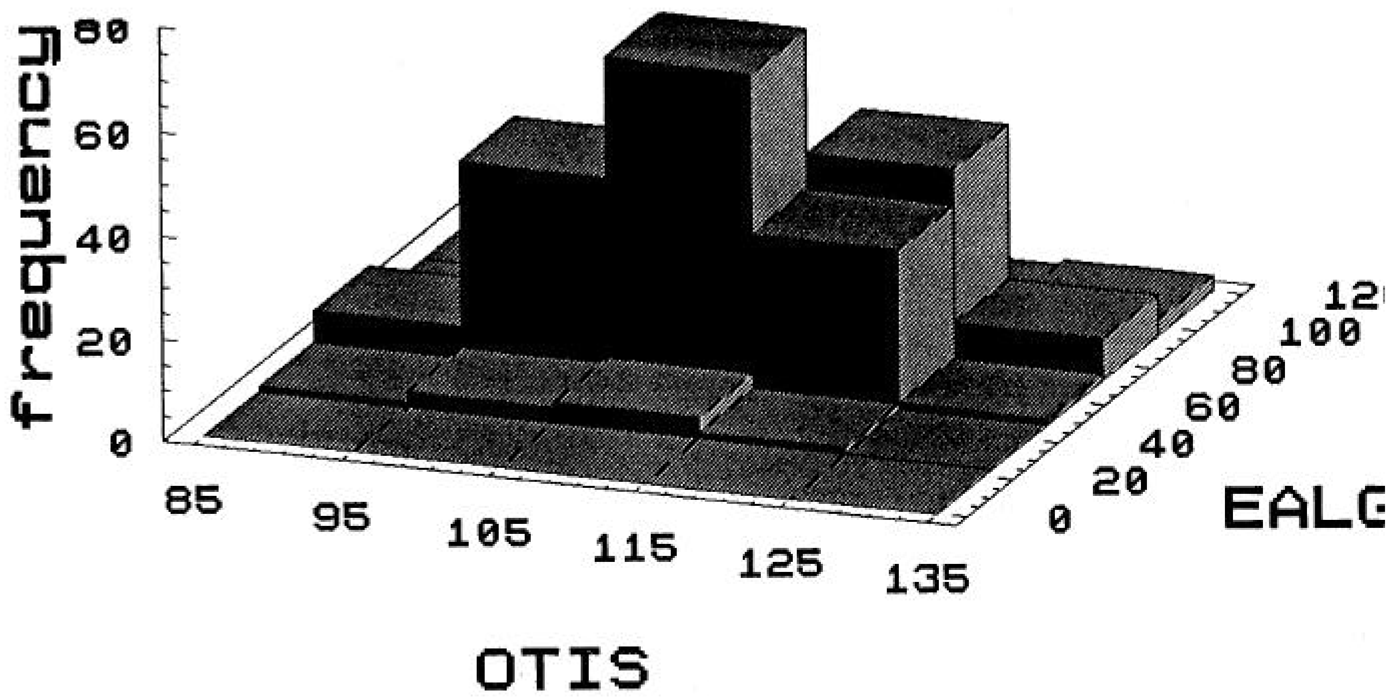
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	87.4639	2.68641	32.5579	.00000
Slope	0.351998	0.0385011	9.14255	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	5053.1081	1	5053.1081	83.586	.00000
Error	14146.197	234	60.454		
Total (Corr.)	19199.305	235			

Correlation Coefficient = 0.513023 R-squared = 26.32 percent
Stnd. Error of Est. = 7.77521

Three-D Histogram of OTIS IQ scores v
Elementary high school algebra score:



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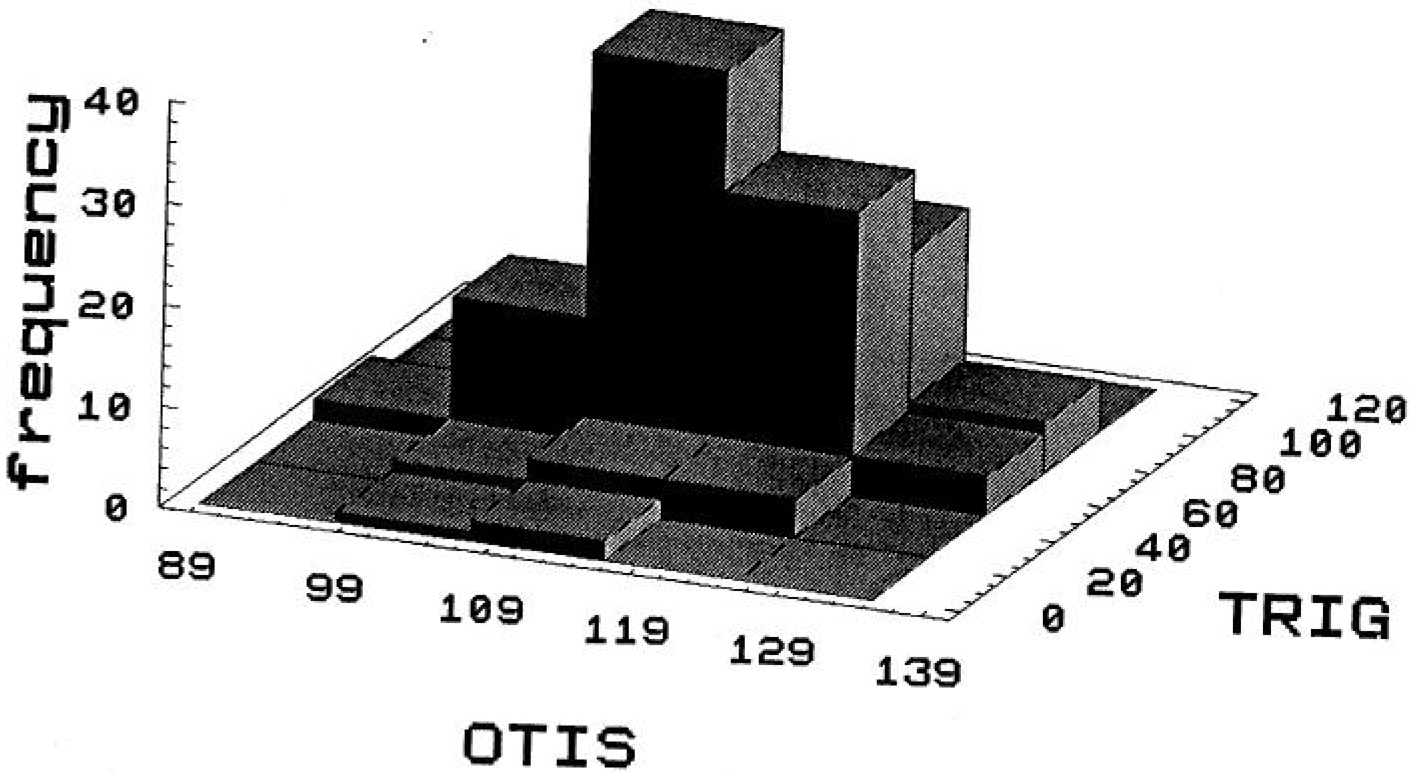
Regression Analysis - Linear model: Y = a+bX
--------------------------------------------------------------------------------
Dependent variable: OTIS
                                                   Independent variable: TR
--------------------------------------------------------------------------------
                                Standard              T               Prob.
Parameter       Estimate          Error             Value              Level
--------------------------------------------------------------------------------
Intercept        105.139         2.63221           39.9433            .00000
Slope            0.14627       0.0400186           3.65506            .00036
--------------------------------------------------------------------------------

                            Analysis of Variance
--------------------------------------------------------------------------------
Source             Sum of Squares     Df  Mean Square    F-Ratio  Prob. Level
Model                   776.85113      1    776.85113    13.3595       .00036
Error                   8373.5872    144      58.1499
--------------------------------------------------------------------------------
Total (Corr.)           9150.4384    145

Correlation Coefficient = 0.291372              R-squared =    8.49 percent
Stnd. Error of Est. = 7.62561
```

Three-D Histogram of OTIS IQ scores vs
High school trigonometry scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: TRIG — Independent variable: OTI

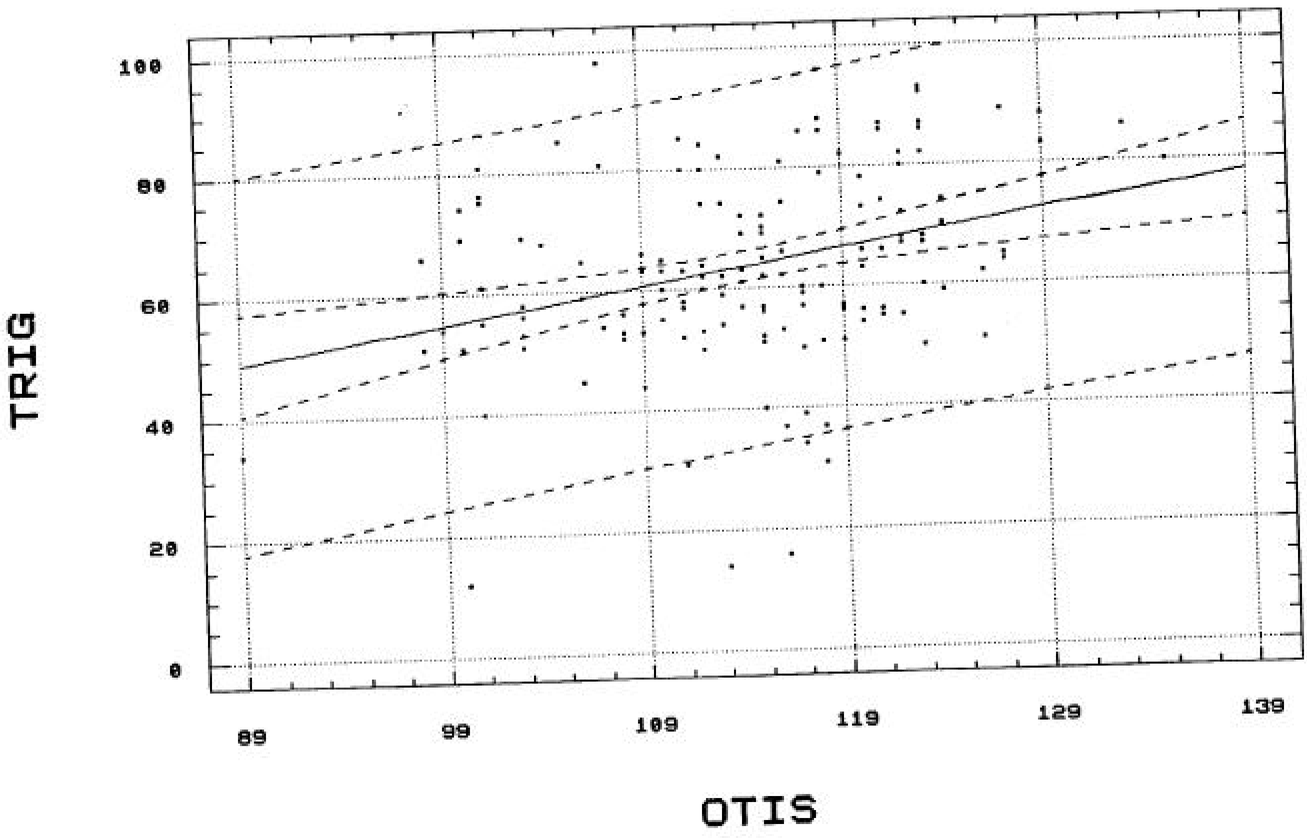
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	-2.58964	18.2226	-0.142112	.88719
Slope	0.580417	0.158798	3.65506	.00036

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	3082.6336	1	3082.6336	13.359	.00036
Error	33227.346	144	230.745		
Total (Corr.)	36309.979	145			

Correlation Coefficient = 0.291372 R-squared = 8.49 percent
Stnd. Error of Est. = 15.1903

Regression of elementary high school TRIGonometry on OTIS IQ scores



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Regression Analysis - Linear model: Y = a+bX
------------------------------------------------------------------------------------
Dependent variable: IALG                                 Independent variable: OT.
------------------------------------------------------------------------------------
                                Standard              T                Prob.
Parameter        Estimate        Error              Value              Level
------------------------------------------------------------------------------------
Intercept        -24.5456        23.7232          -1.03467            .30429
Slope            0.795408        0.206123           3.8589            .00025
------------------------------------------------------------------------------------

                           Analysis of Variance
------------------------------------------------------------------------------------
Source               Sum of Squares     Df   Mean Square     F-Ratio   Prob. Level
Model                     2738.2448      1     2738.2448      14.891        .00025
Error                     13239.715     72       183.885
------------------------------------------------------------------------------------
Total (Corr.)             15977.959     73

Correlation Coefficient = 0.413976             R-squared =  17.14 percent
Stnd. Error of Est. = 13.5604
```
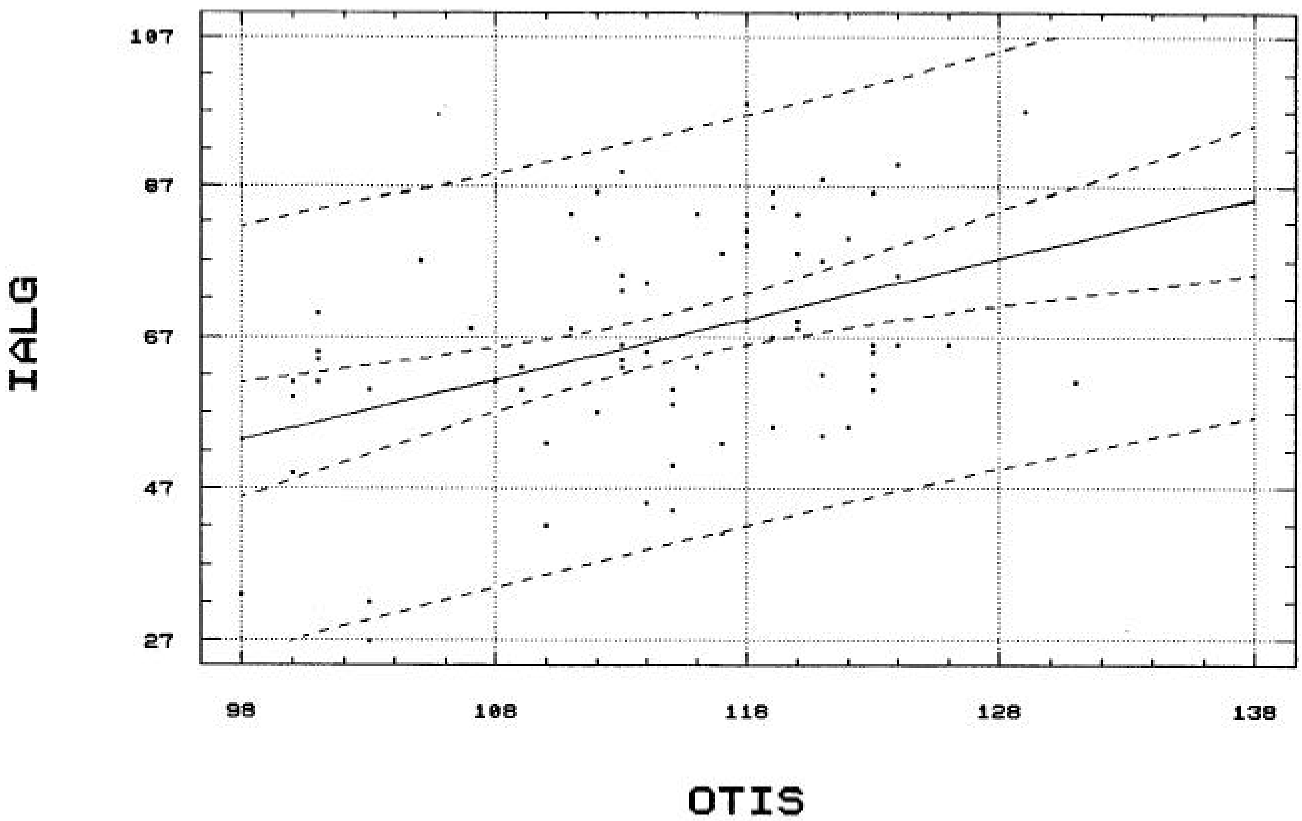

Regression of high school intermediate algebra scores on OTIS IQ scores



Regression Analysis - Linear model: Y = a+bX

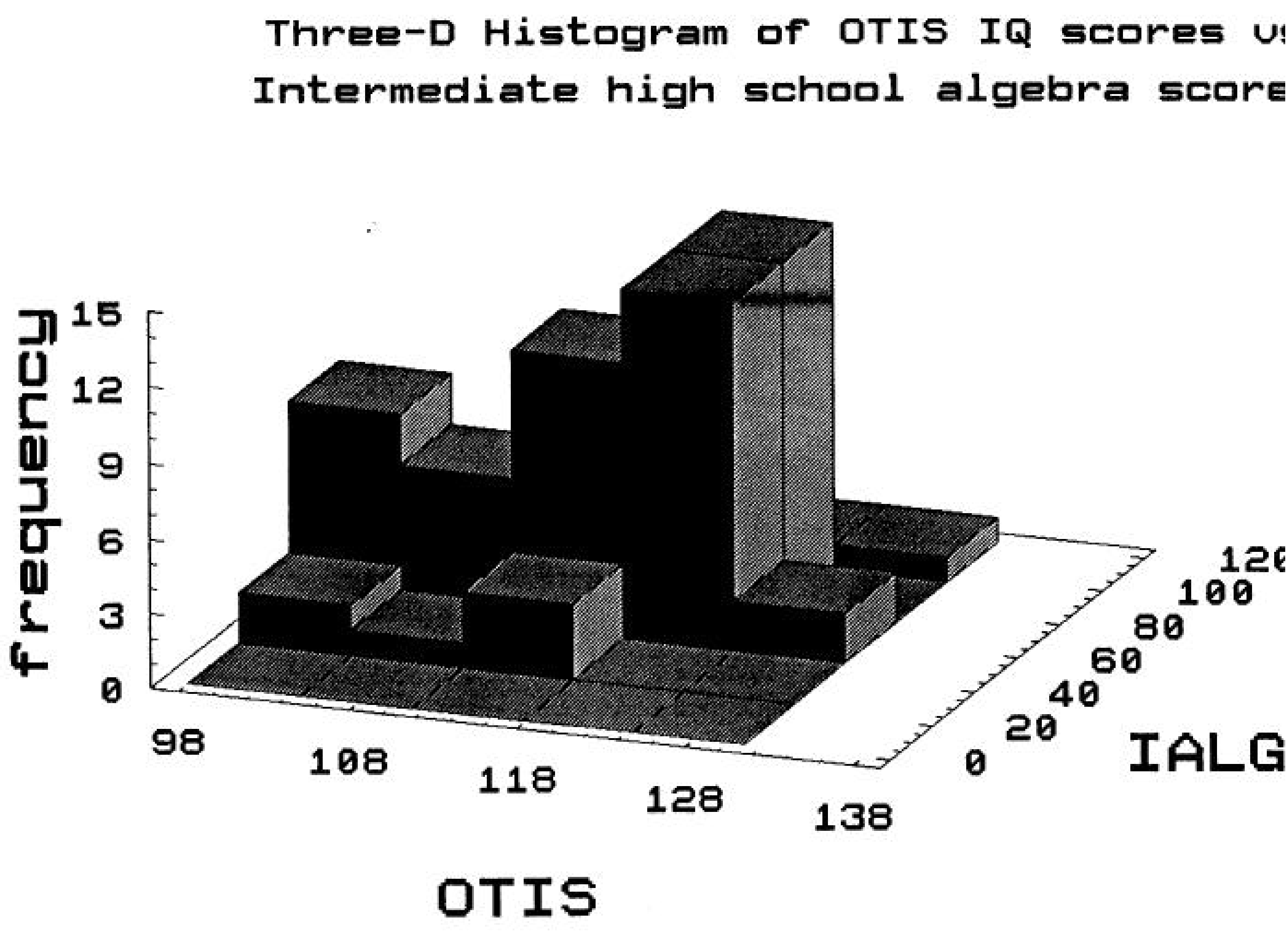
Dependent variable: OTIS Independent variable: IAL

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	100.446	3.81873	26.3035	.00000
Slope	0.215457	0.0558339	3.8589	.00025

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	741.72622	1	741.72622	14.8911	.00025
Error	3586.3278	72	49.8101		
Total (Corr.)	4328.0541	73			

Correlation Coefficient = 0.413976 R-squared = 17.14 percent
Stnd. Error of Est. = 7.05763



Regression Analysis - Linear model: Y = a+bX

Dependent variable: OTIS   Independent variable: ANA

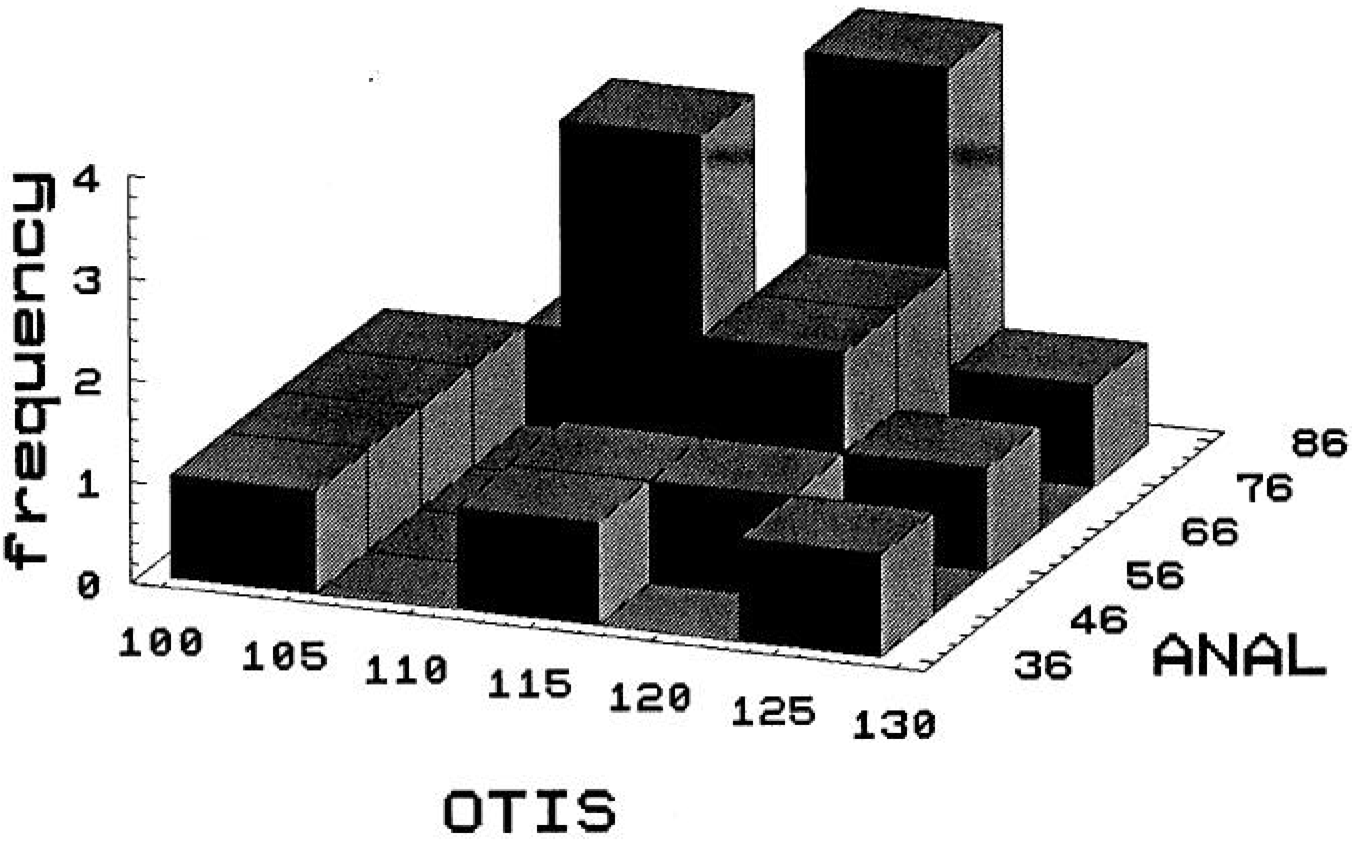
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	103.991	7.43853	13.98	.00000
Slope	0.174838	0.116949	1.495	.14743

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	132.17761	1	132.17761	2.2350	.14743
Error	1478.4891	25	59.1396		
Total (Corr.)	1610.6667	26			

Correlation Coefficient = 0.286468   R-squared = 8.21 percent
Stnd. Error of Est. = 7.69023

Three-D Histogram of OTIS IQ scores v
High school analytic geometry scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: ANAL   Independent variable: OTIS

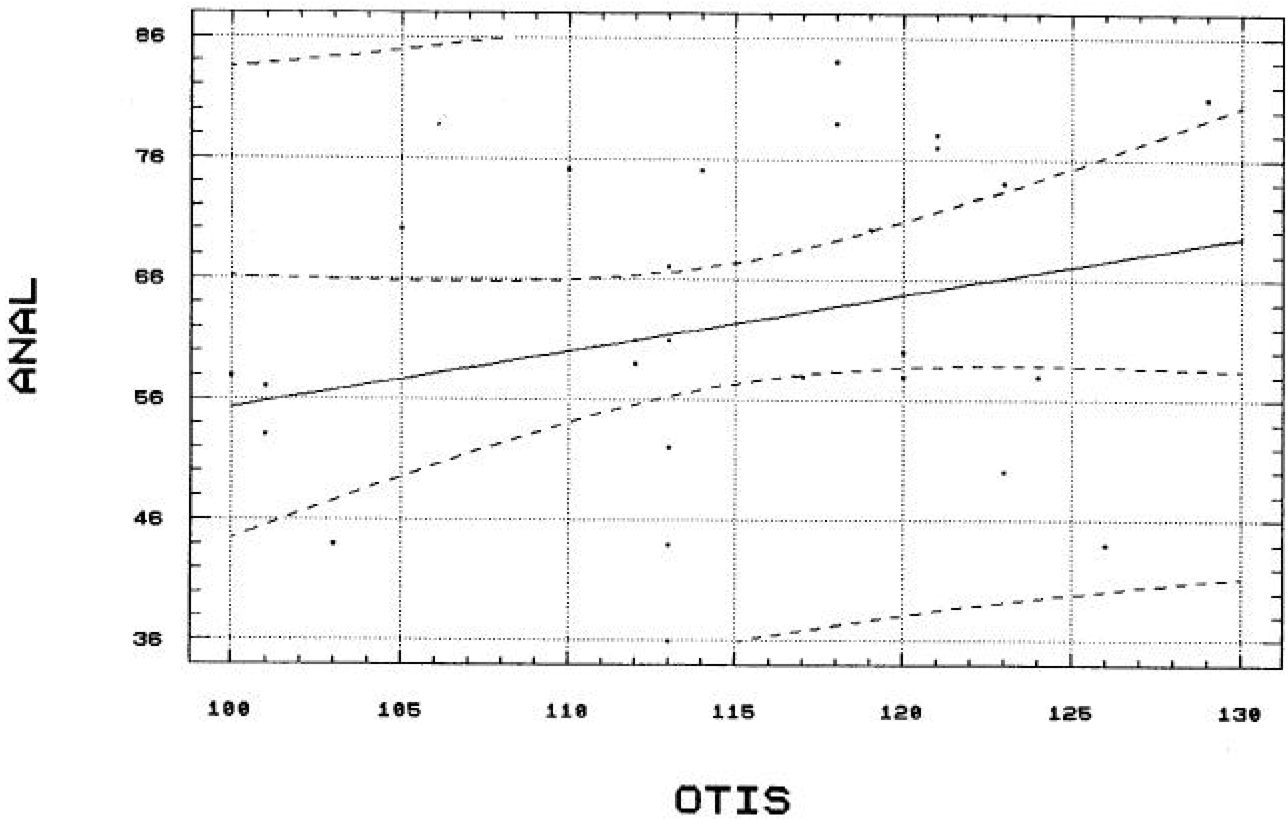
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	8.40784	36.1521	0.232568	.81799
Slope	0.469371	0.313961	1.495	.14743

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	354.84437	1	354.84437	2.2350	.14743
Error	3969.1556	25	158.7662		
Total (Corr.)	4324.0000	26			

Correlation Coefficient = 0.286468   R-squared =   8.21 percent
Stnd. Error of Est. = 12.6002

Regression of high school ANALytic geometry scores on OTIS IQ scores



egression Analysis - Linear model: Y = a+bX

ependent variable: PSATM Independent variable: PSATV

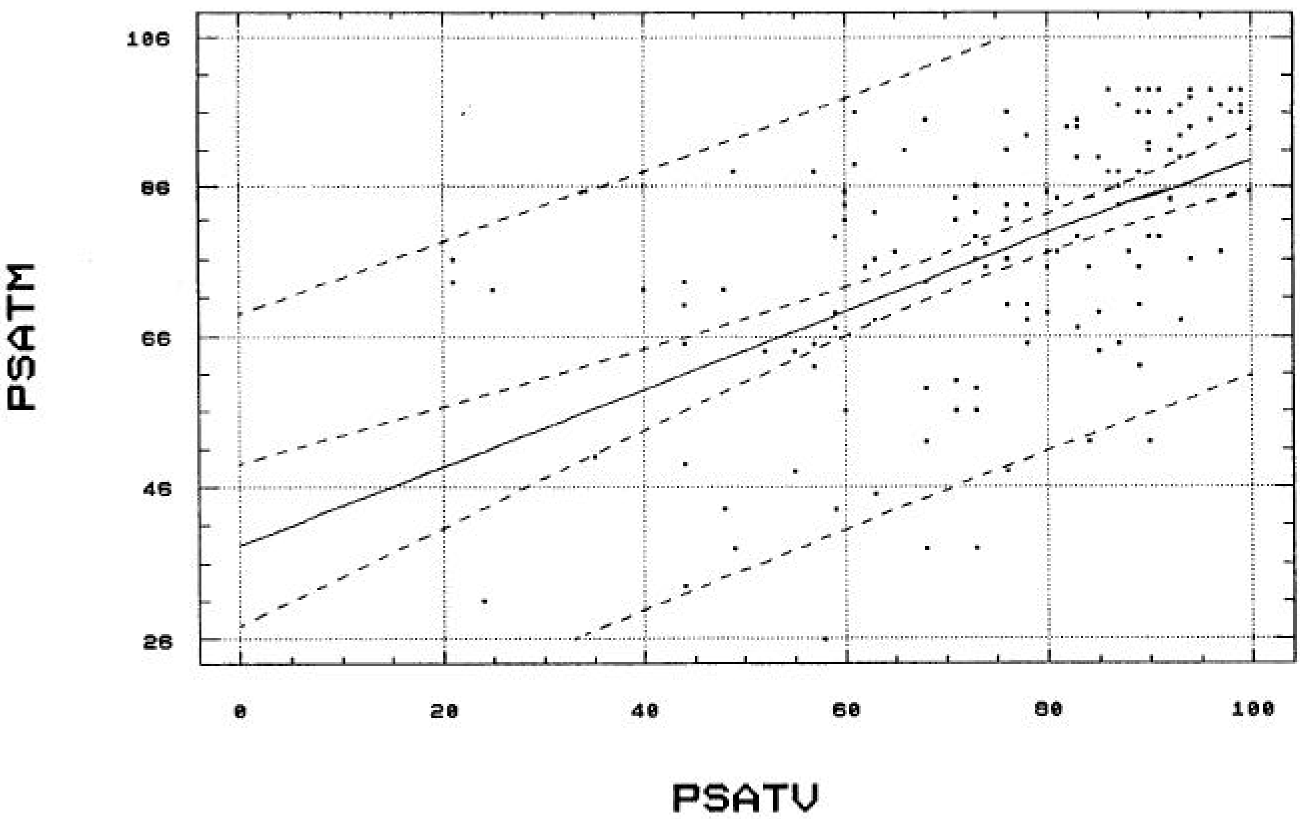
'arameter	Estimate	Standard Error	T Value	Prob. Level
:ntercept	38.3723	5.39461	7.11307	.00000
;lope	0.512777	0.0706072	7.26239	.00000

Analysis of Variance

;ource	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
1odel	10996.342	1	10996.342	52.74	.00000
:rror	27729.406	133	208.492		
:otal (Corr.)	38725.748	134			

:orrelation Coefficient = 0.532874 R-squared = 28.40 percent
;tnd. Error of Est. = 14.4392

Regression of PSAT scores in mathematics on PSAT verbal scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: PSATV | Independent variable: PSATM

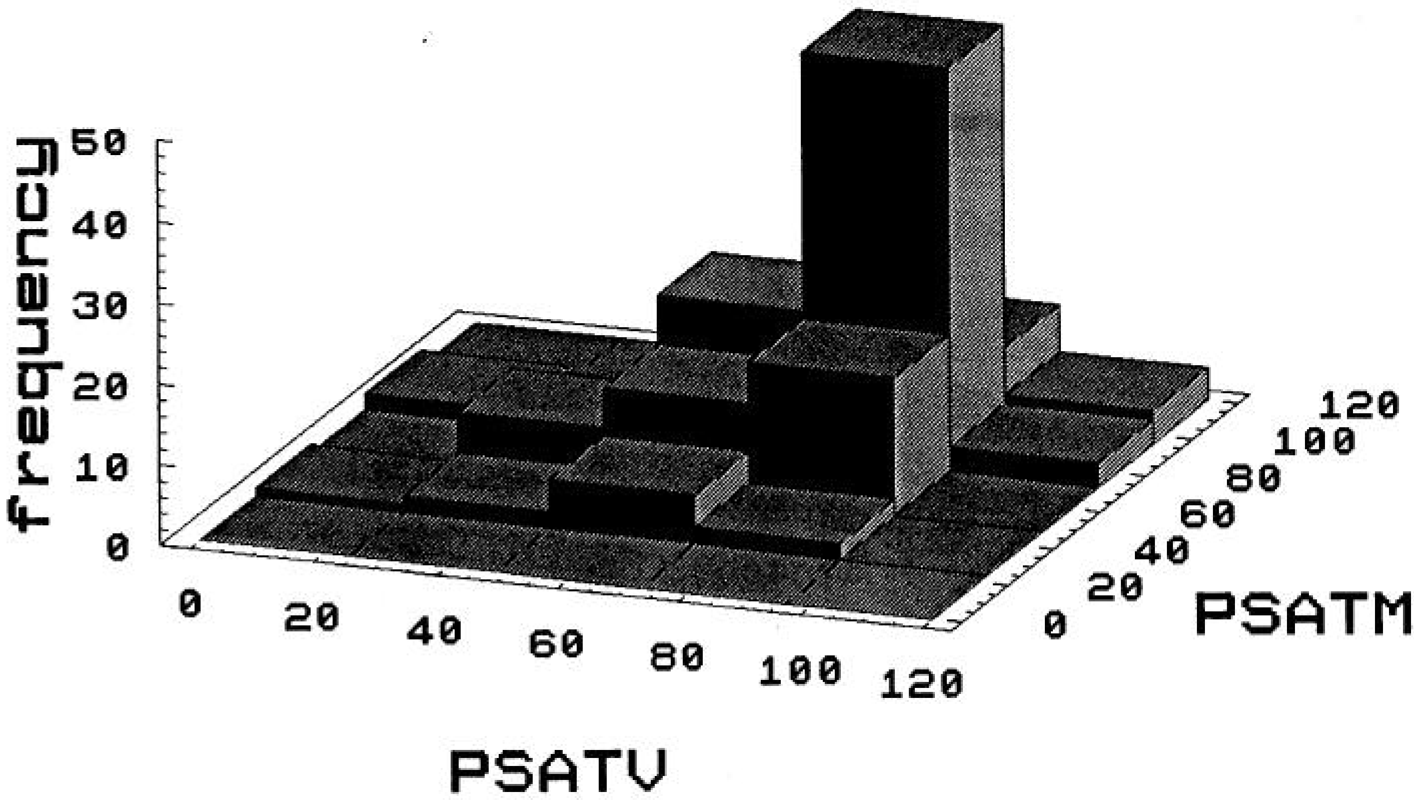
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	31.9877	5.9741	5.3544	.00000
Slope	0.553758	0.07625	7.26239	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	11875.149	1	11875.149	52.74	.00000
Error	29945.488	133	225.154		
Total (Corr.)	41820.637	134			

Correlation Coefficient = 0.532874   R-squared = 28.40 percent
Stnd. Error of Est. = 15.0051

Three-D Histogram of PSAT verbal scores vs PSAT mathematical aptitude scores



egression Analysis - Linear model: Y = a+bX

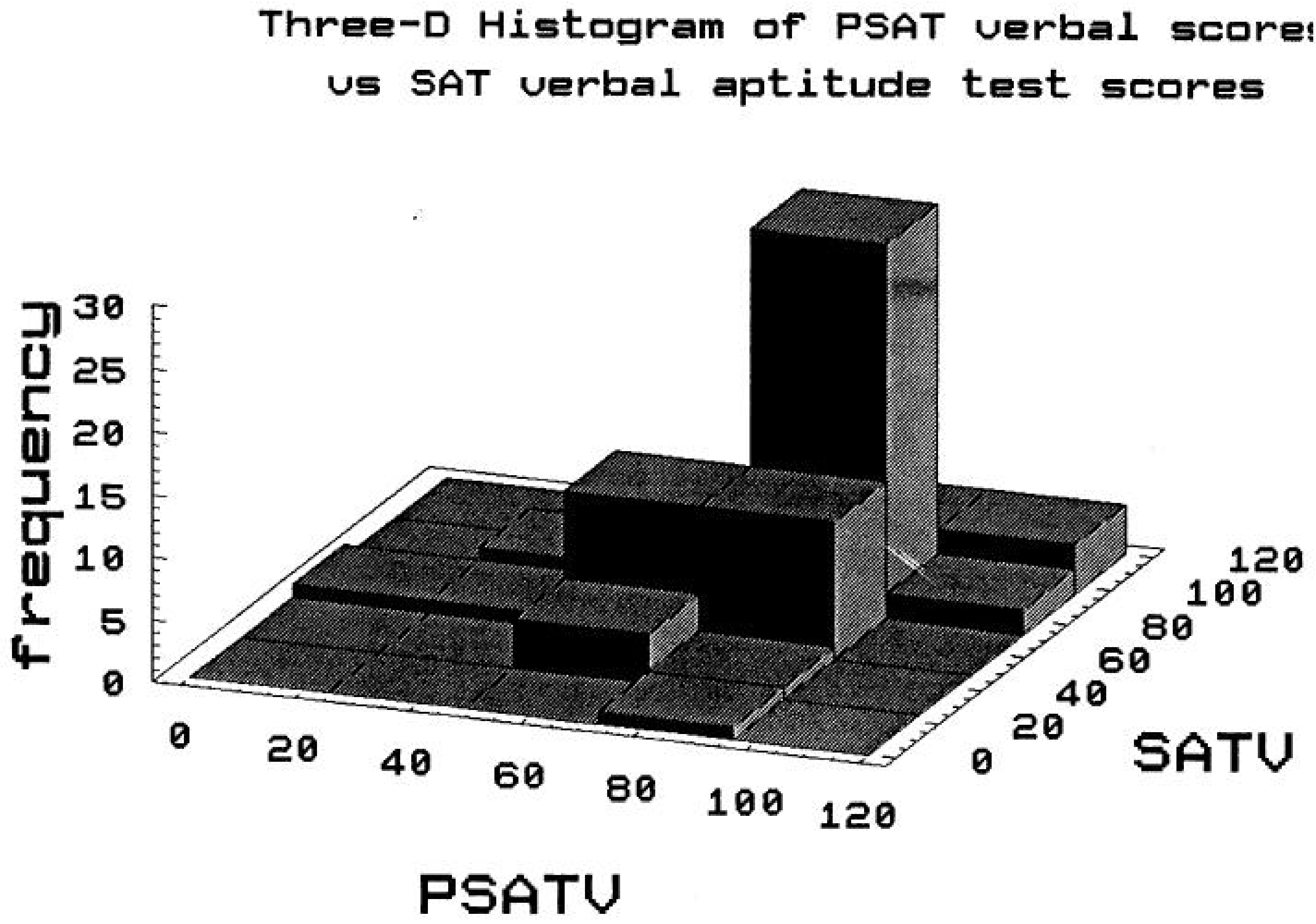
ependent variable: PSATV — Independent variable: SATV

arameter	Estimate	Standard Error	T Value	Prob. Level
ntercept	40.3644	5.80567	6.95259	.00000
lope	0.489774	0.0750444	6.52645	.00000

Analysis of Variance

ource	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
odel	6887.3742	1	6887.3742	42.595	.00000
rror	11965.507	74	161.696		
otal (Corr.)	18852.882	75			

orrelation Coefficient = 0.604419 R-squared = 36.53 percent
tnd. Error of Est. = 12.716



Regression Analysis - Linear model: Y = a+bX

Dependent variable: SATV

Independent variable: PSATV

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	17.4179	8.98686	1.93815	.05642
Slope	0.7459	0.114289	6.52645	.00000

Analysis of Variance

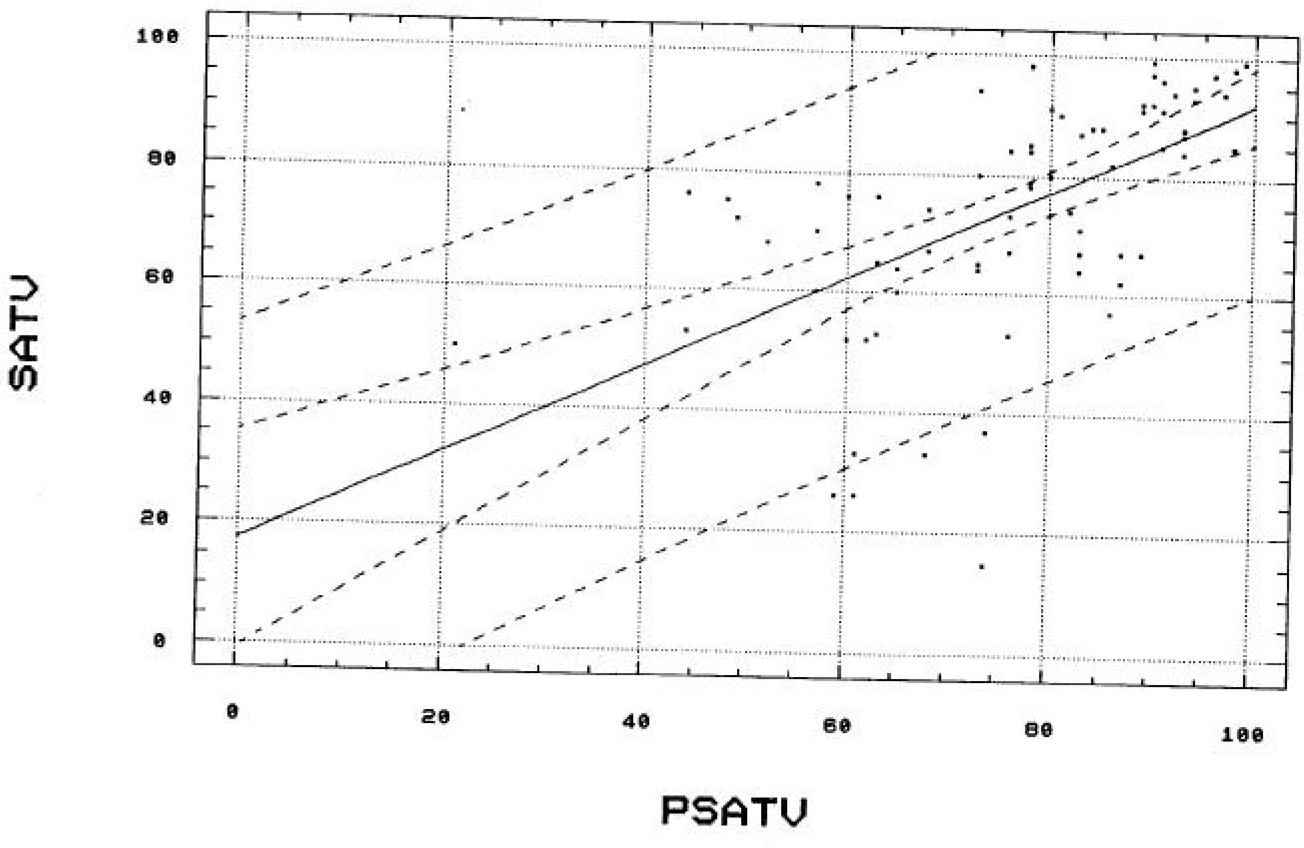
Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	10489.104	1	10489.104	42.59	.00000
Error	18222.830	74	246.254		
Total (Corr.)	28711.934	75			

Correlation Coefficient = 0.604419

R-squared = 36.53 percent

Stnd. Error of Est. = 15.6925

Regression of SATV verbal aptitude scores on PSAT verbal aptitude scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: SATM — Independent variable: PSATV

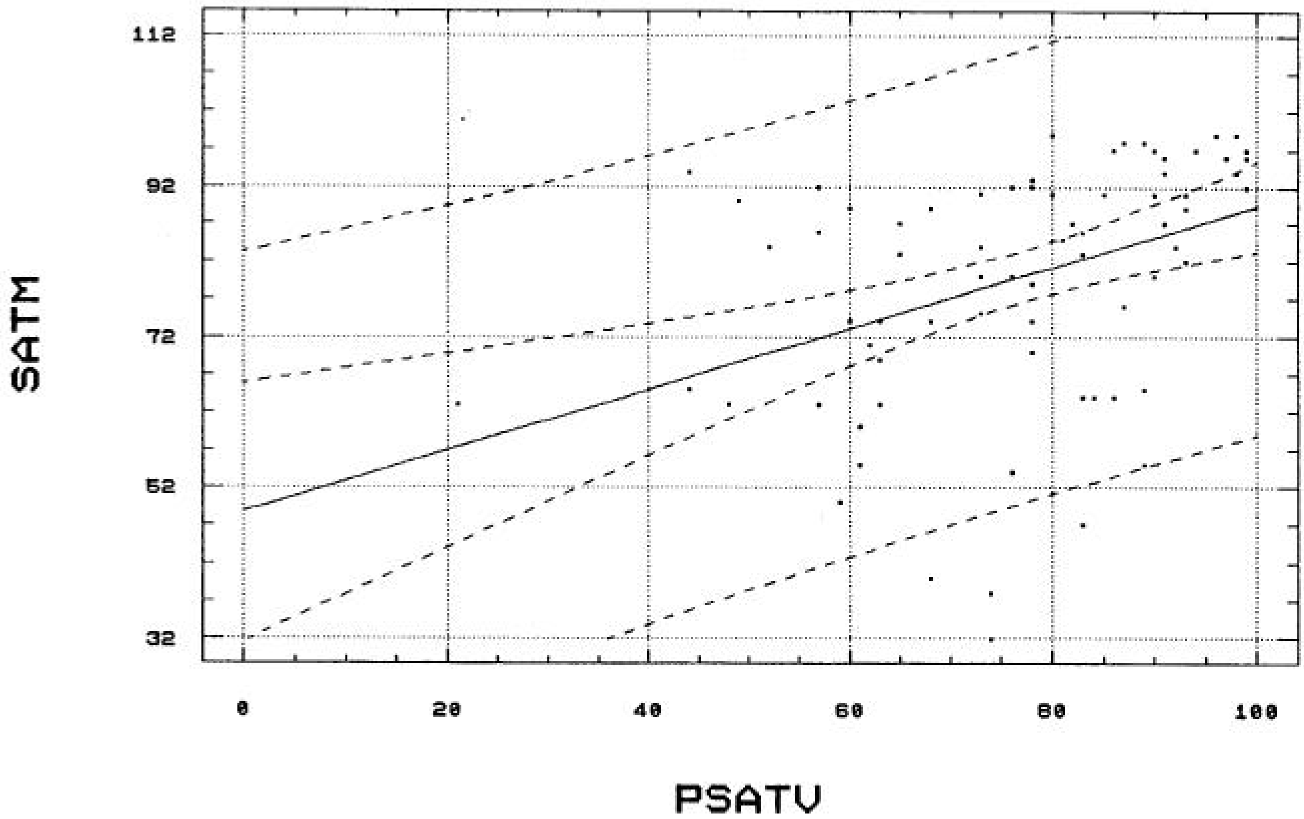
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	48.8028	8.58438	5.68508	.00000
Slope	0.405292	0.10917	3.71248	.00040

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	3096.8038	1	3096.8038	13.782	.00040
Error	16627.144	74	224.691		
Total (Corr.)	19723.947	75			

Correlation Coefficient = 0.396241 R-squared = 15.70 percent
Stnd. Error of Est. = 14.9897

Regression of SAT scores in mathematics on PSAT verbal aptitude scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: PSATV Independent variable: SATM

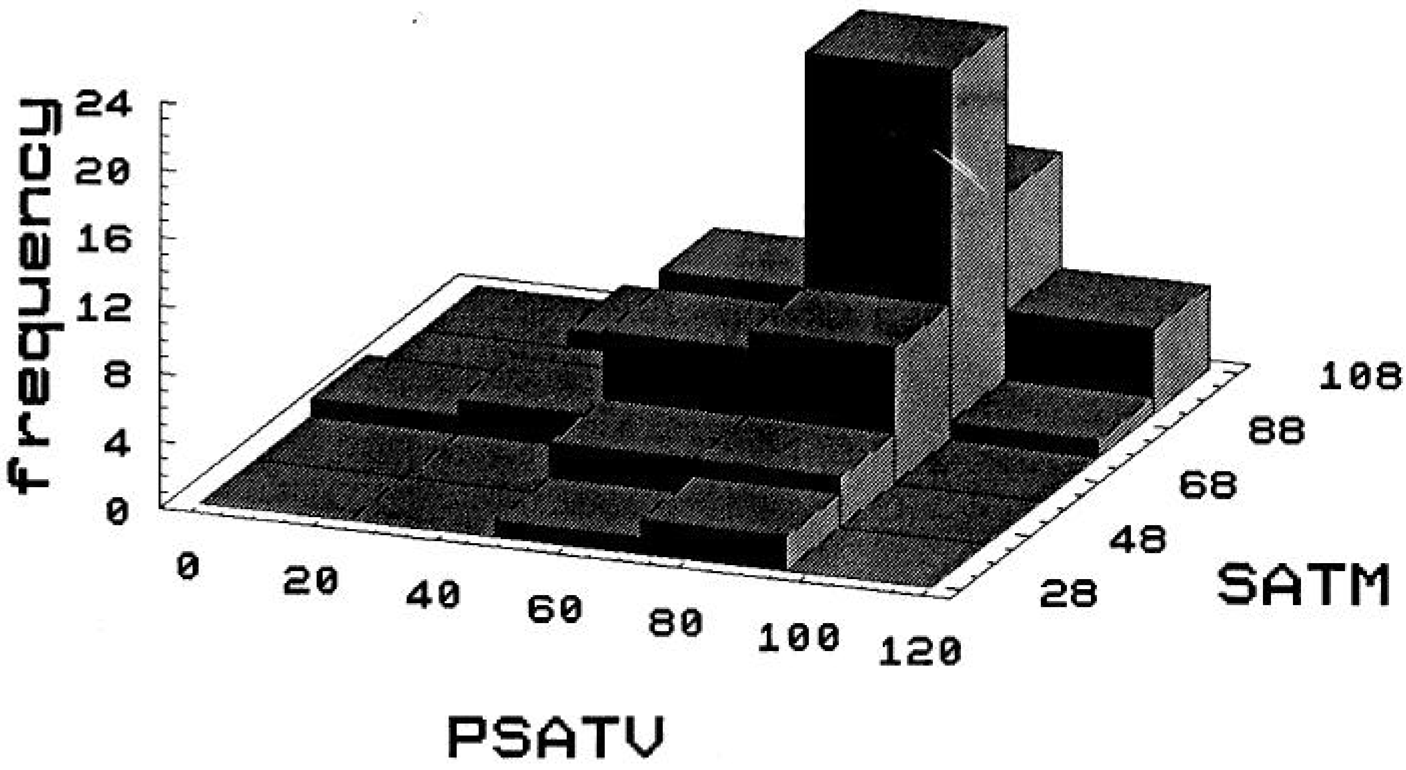
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	46.0378	8.51818	5.40465	.00000
Slope	0.387393	0.104349	3.71248	.00040

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	2960.0401	1	2960.0401	13.782	.00040
Error	15892.842	74	214.768		
Total (Corr.)	18852.882	75			

Correlation Coefficient = 0.396241 R-squared = 15.70 percent
Stnd. Error of Est. = 14.655

Three-D Histogram of PSAT verbal scores vs SAT mathematical aptitude test scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: PSATV | Independent variable: GEOM

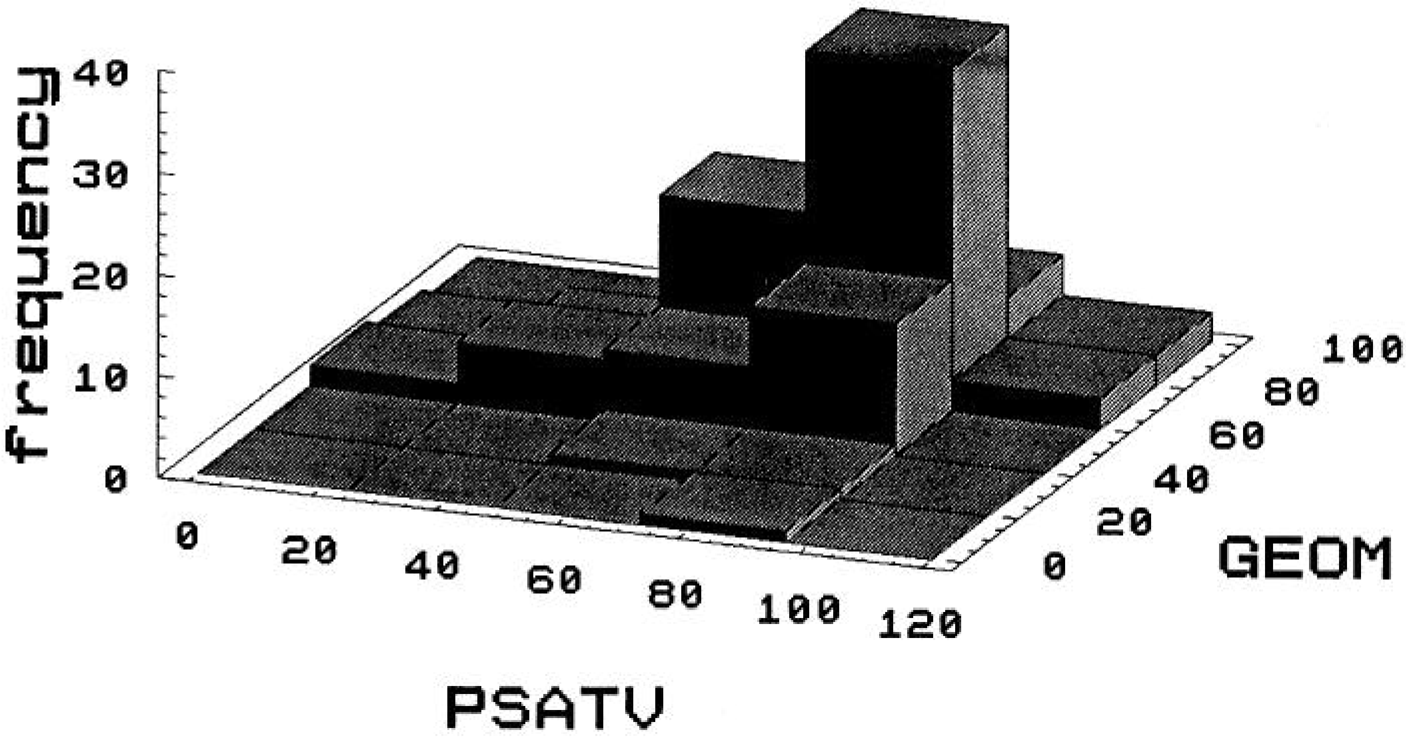
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	40.1979	9.2987	4.32296	.00004
Slope	0.489111	0.137647	3.55336	.00058

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	3957.5670	1	3957.5670	12.626	.00058
Error	31343.727	100	313.437		
Total (Corr.)	35301.294	101			

Correlation Coefficient = 0.334826 R-squared = 11.21 percent
Stnd. Error of Est. = 17.7042

Three-D Histogram of PSAT verbal scores vs High school geometry scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: GEOM Independent variable: PSAT

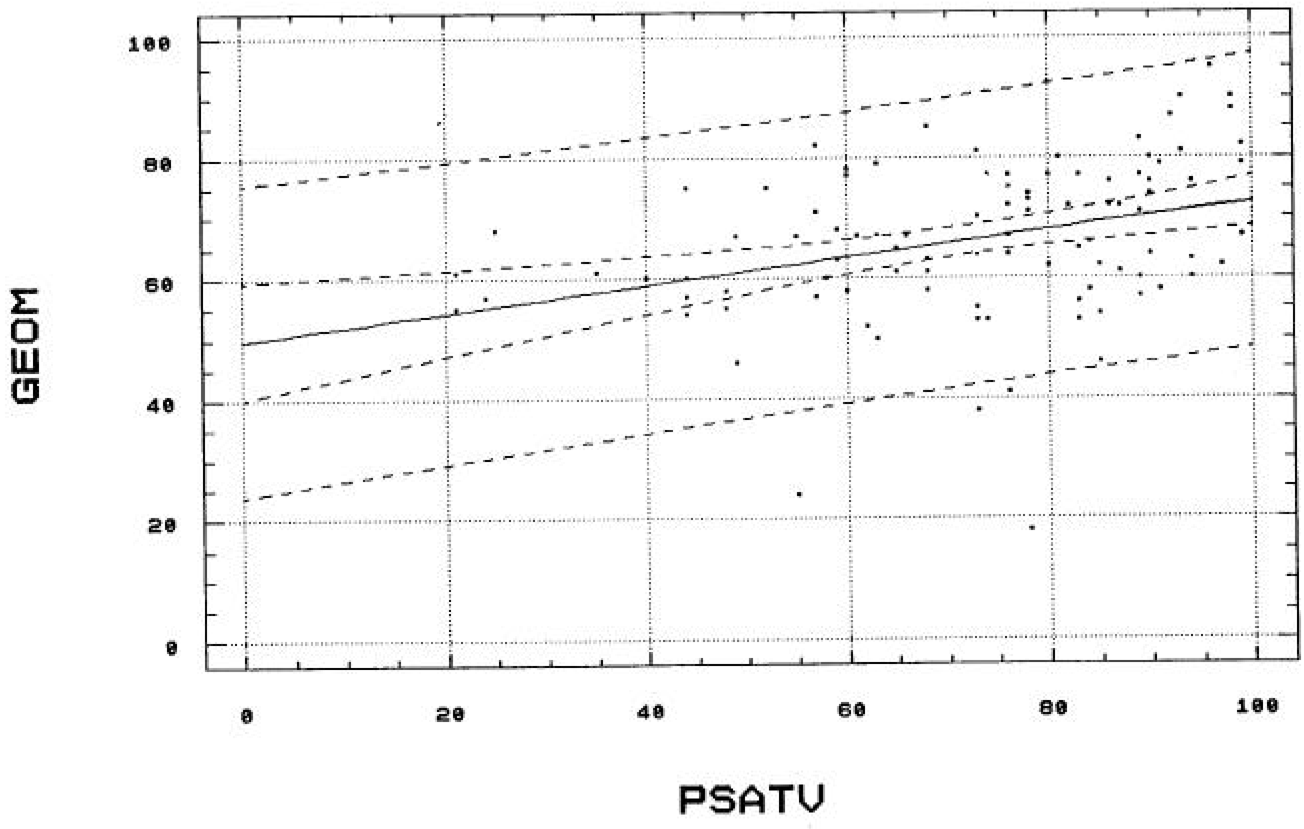
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	49.6918	4.83729	10.2727	.00000
Slope	0.229208	0.0645048	3.55336	.00058

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	1854.6060	1	1854.6060	12.626	.00058
Error	14688.384	100	146.884		
Total (Corr.)	16542.990	101			

Correlation Coefficient = 0.334826 R-squared = 11.21 percent
Stnd. Error of Est. = 12.1196

Regression of high school GEOMetry scores on PSAT verbal aptitude scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: EALG | Independent variable: PSATV

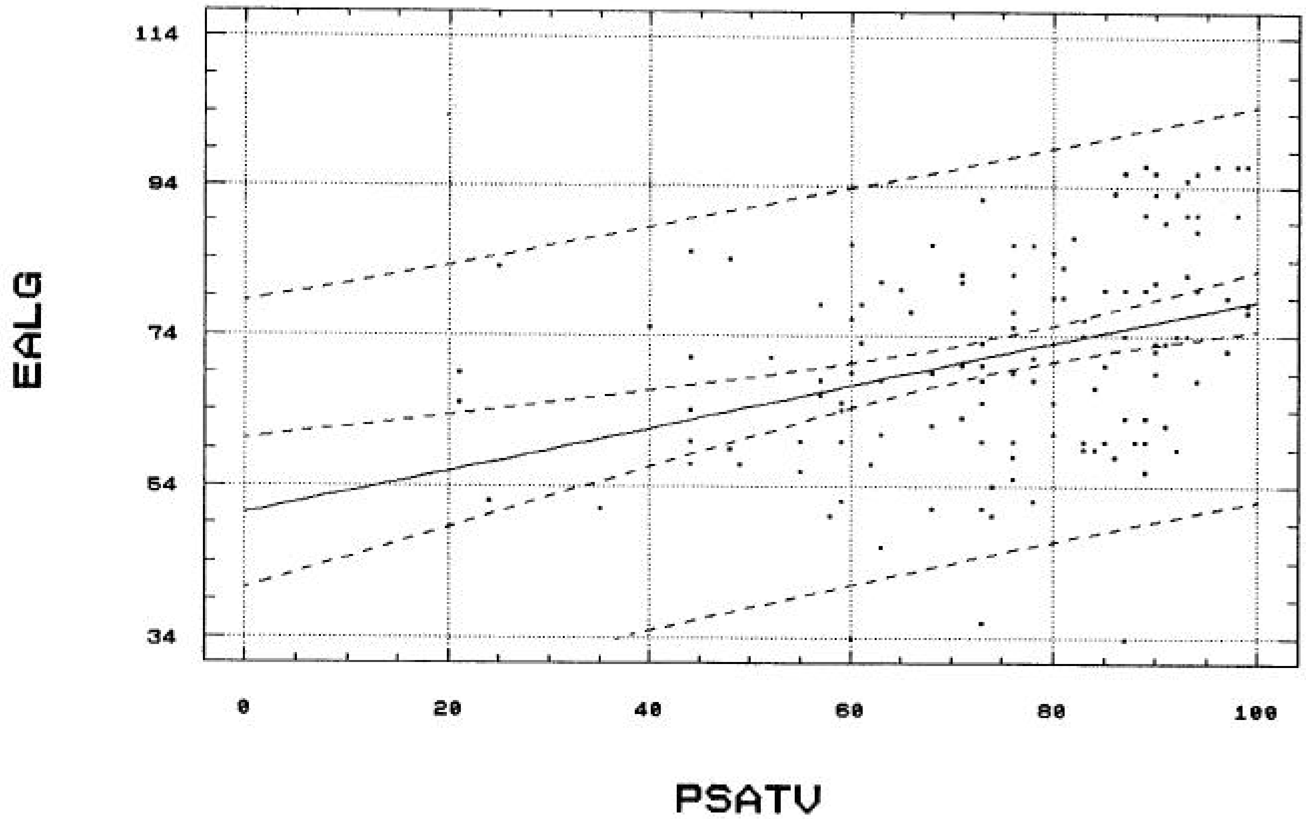
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	50.3609	5.00162	10.0689	.00000
Slope	0.28213	0.0652906	4.32114	.00003

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	3264.1711	1	3264.1711	18.672	.00003
Error	22550.974	129	174.814		
Total (Corr.)	25815.145	130			

Correlation Coefficient = 0.35559 R-squared = 12.64 percent
Stnd. Error of Est. = 13.2217

Regression of high school algebra scores on PSAT verbal aptitude scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: PSATV | Independent variable: EALG

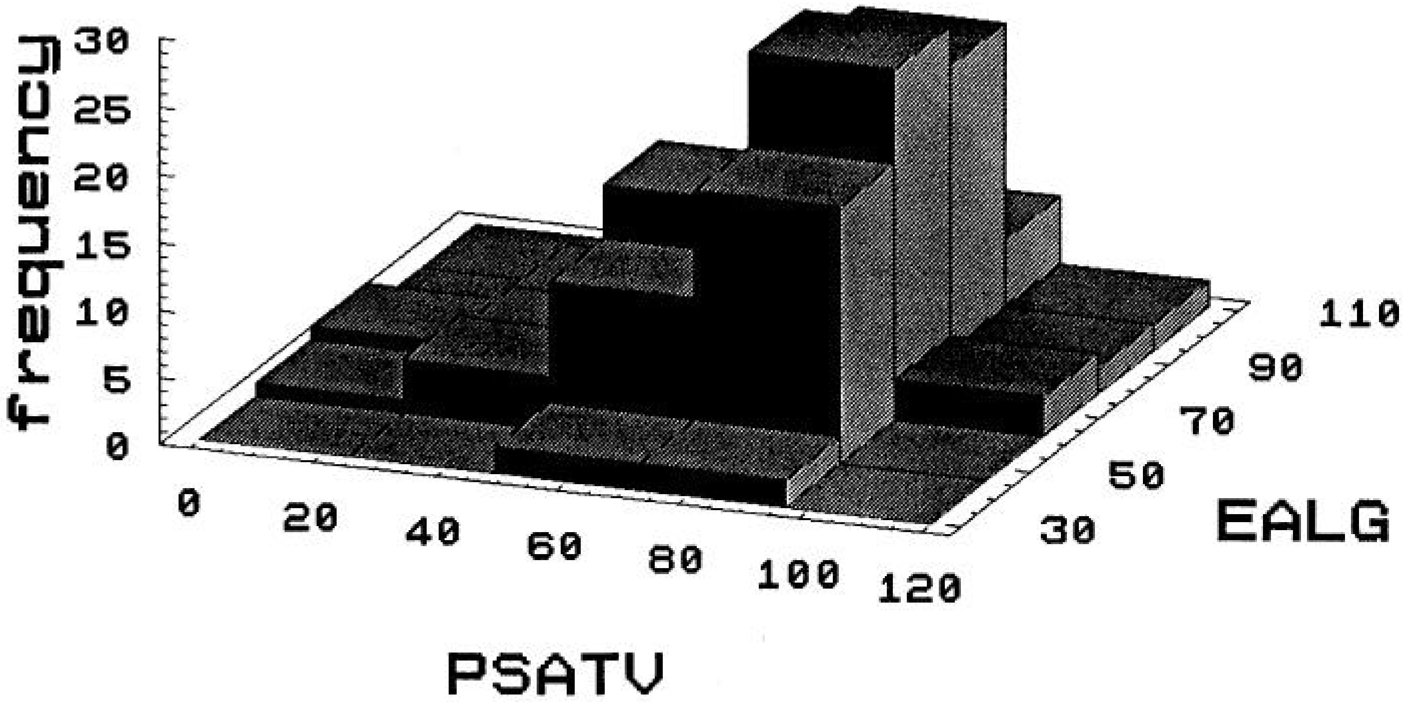
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	42.5393	7.54609	5.63727	.00000
Slope	0.448177	0.103717	4.32114	.00003

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	5185.2922	1	5185.2922	18.672	.00003
Error	35823.303	129	277.700		
Total (Corr.)	41008.595	130			

Correlation Coefficient = 0.35559 R-squared = 12.64 percent
Stnd. Error of Est. = 16.6643

Three-D Histogram of PSAT verbal scores vs High school elementary algebra scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: PSATV Independent variable: TRIG

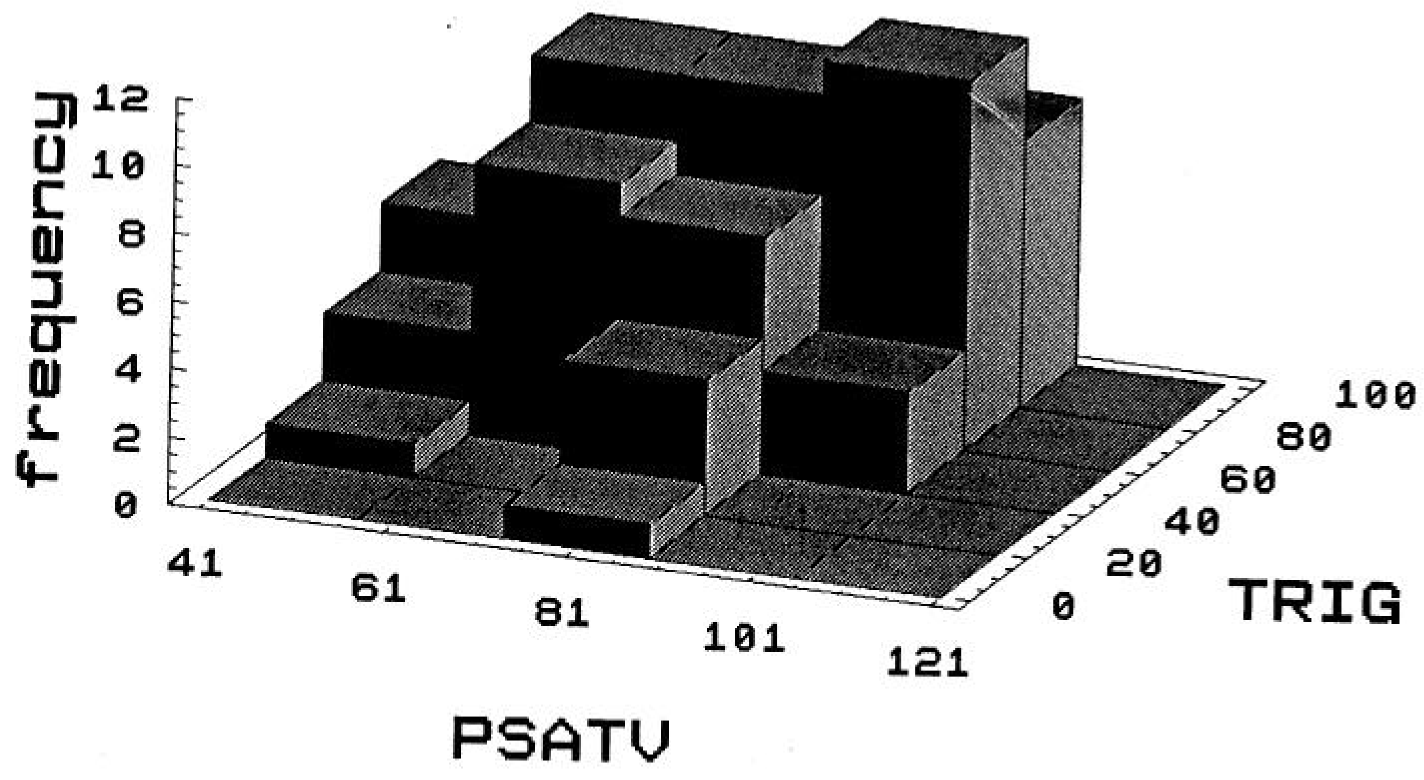
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	60.5539	7.18676	8.42575	.00000
Slope	0.255748	0.106232	2.40745	.01849

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	1186.6370	1	1186.6370	5.796	.01849
Error	15560.235	76	204.740		
Total (Corr.)	16746.872	77			

Correlation Coefficient = 0.26619 R-squared = 7.09 percent
Stnd. Error of Est. = 14.3087

Three-D Histogram of PSAT verbal scores vs High school trigonometry scores



Regression Analysis - Linear model: Y = a+bX

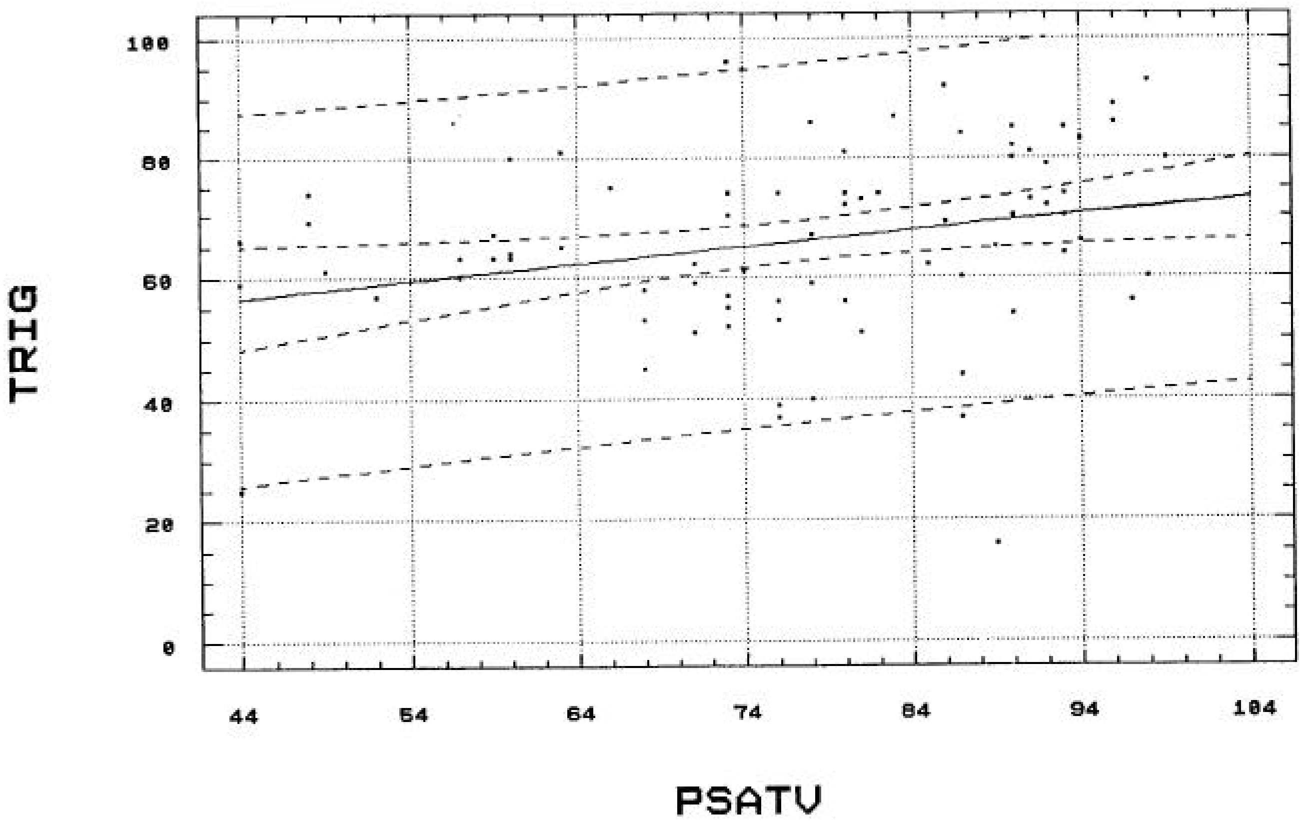
Dependent variable: TRIG			Independent variable: PSATV	
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	44.463	9.06687	4.9039	.00001
Slope	0.277059	0.115084	2.40745	.01849

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	1285.5183	1	1285.5183	5.796	.01849
Error	16856.853	76	221.801		
Total (Corr.)	18142.372	77			

Correlation Coefficient = 0.26619 R-squared = 7.09 percent
Stnd. Error of Est. = 14.893

Regression of high school TRIGonometry scores on PSAT verbal aptitude scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: IALG	Independent variable: PSATV

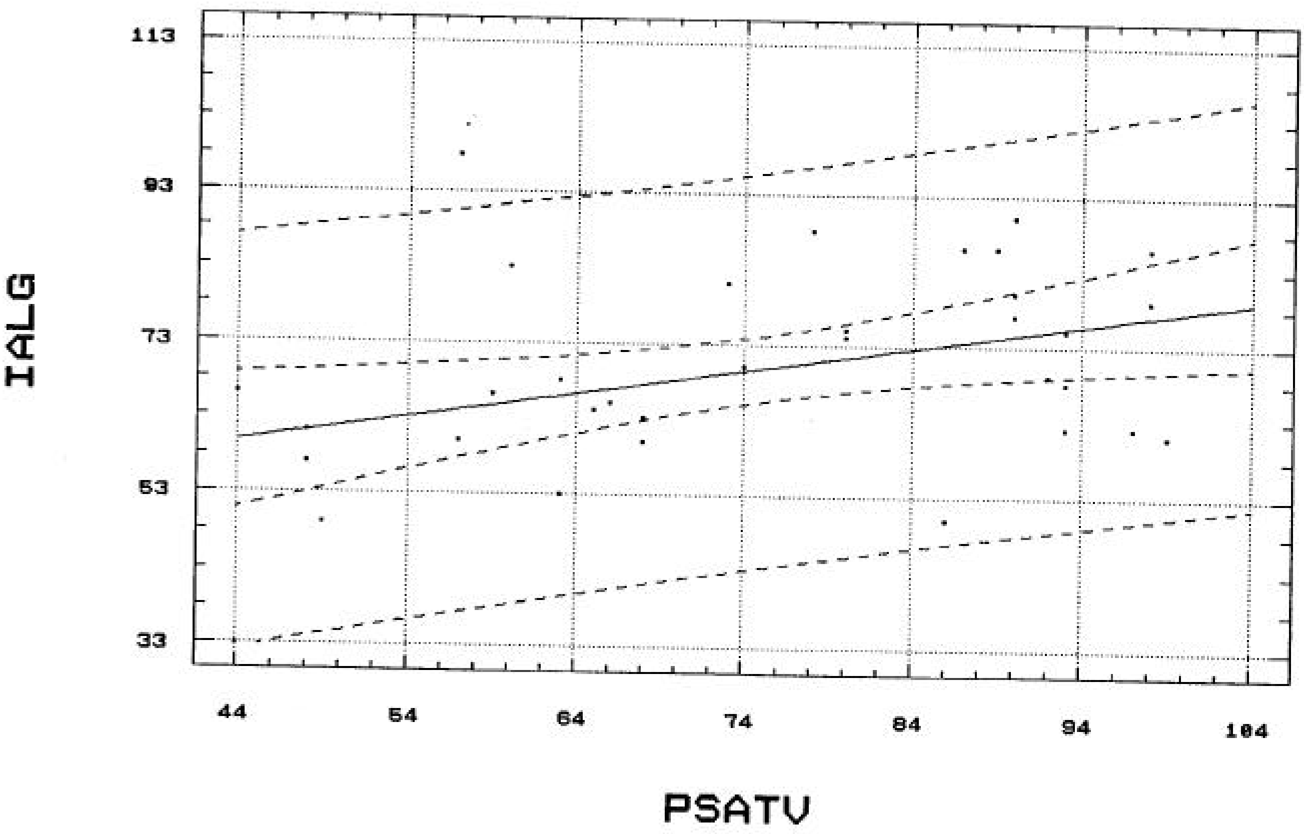
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	45.6452	9.53107	4.7891	.00004
Slope	0.320229	0.124291	2.57644	.01480

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	1059.0447	1	1059.0447	6.638	.01480
Error	5105.3377	32	159.5418		
Total (Corr.)	6164.3824	33			

Correlation Coefficient = 0.414488　　R-squared = 17.18 percent
Stnd. Error of Est. = 12.631

Regression of intermediate algebra scores on PSAT verbal aptitude scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: PSATV   Independent variable: IAL

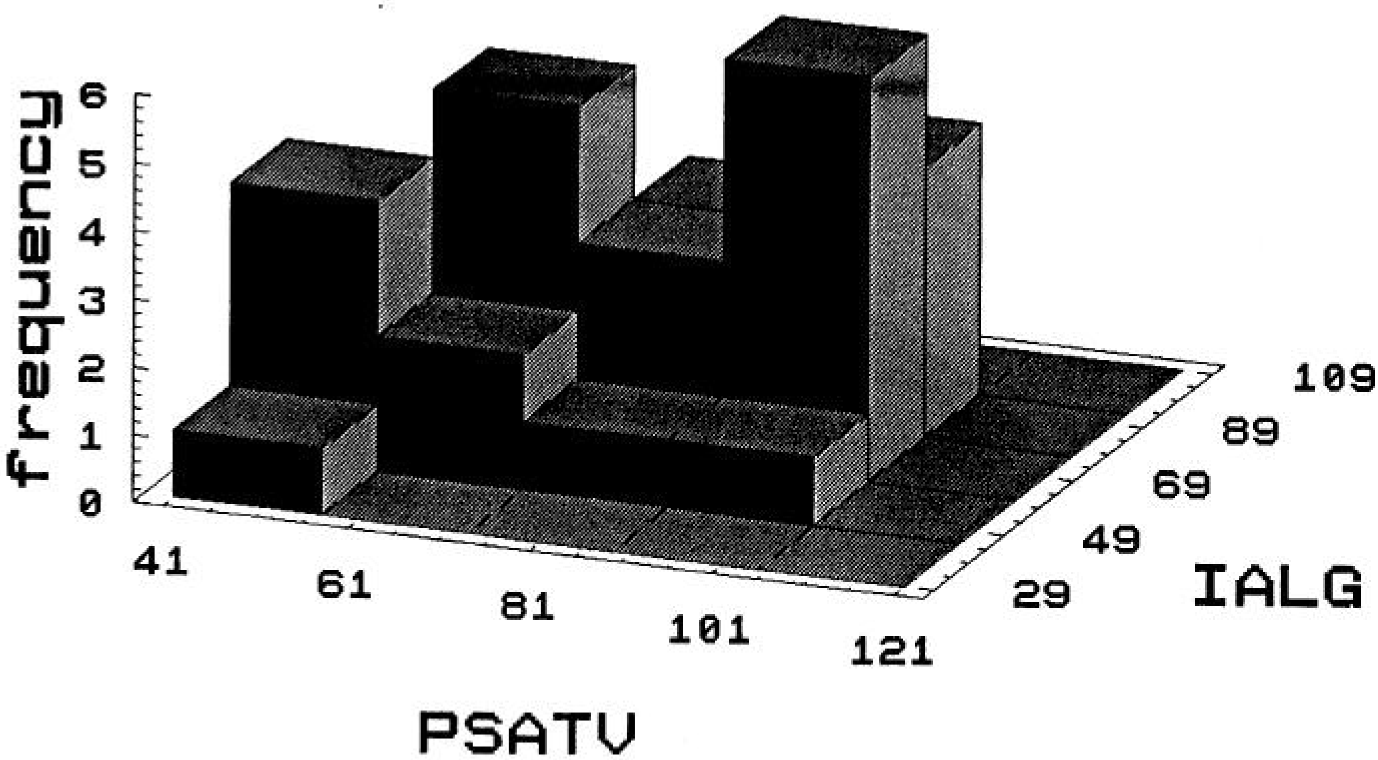
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	37.3587	14.7531	2.53225	.01644
Slope	0.536493	0.20823	2.57644	.01480

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	1774.2608	1	1774.2608	6.638	.01480
Error	8553.1804	32	267.2869		
Total (Corr.)	10327.441	33			

Correlation Coefficient = 0.414488   R-squared = 17.18 percent
Stnd. Error of Est. = 16.3489

Three-D Histogram of PSAT verbal scores vs Intermediate algebra scores



gression Analysis - Linear model: Y = a+bX

pendent variable: PSATV   Independent variable: ANAL

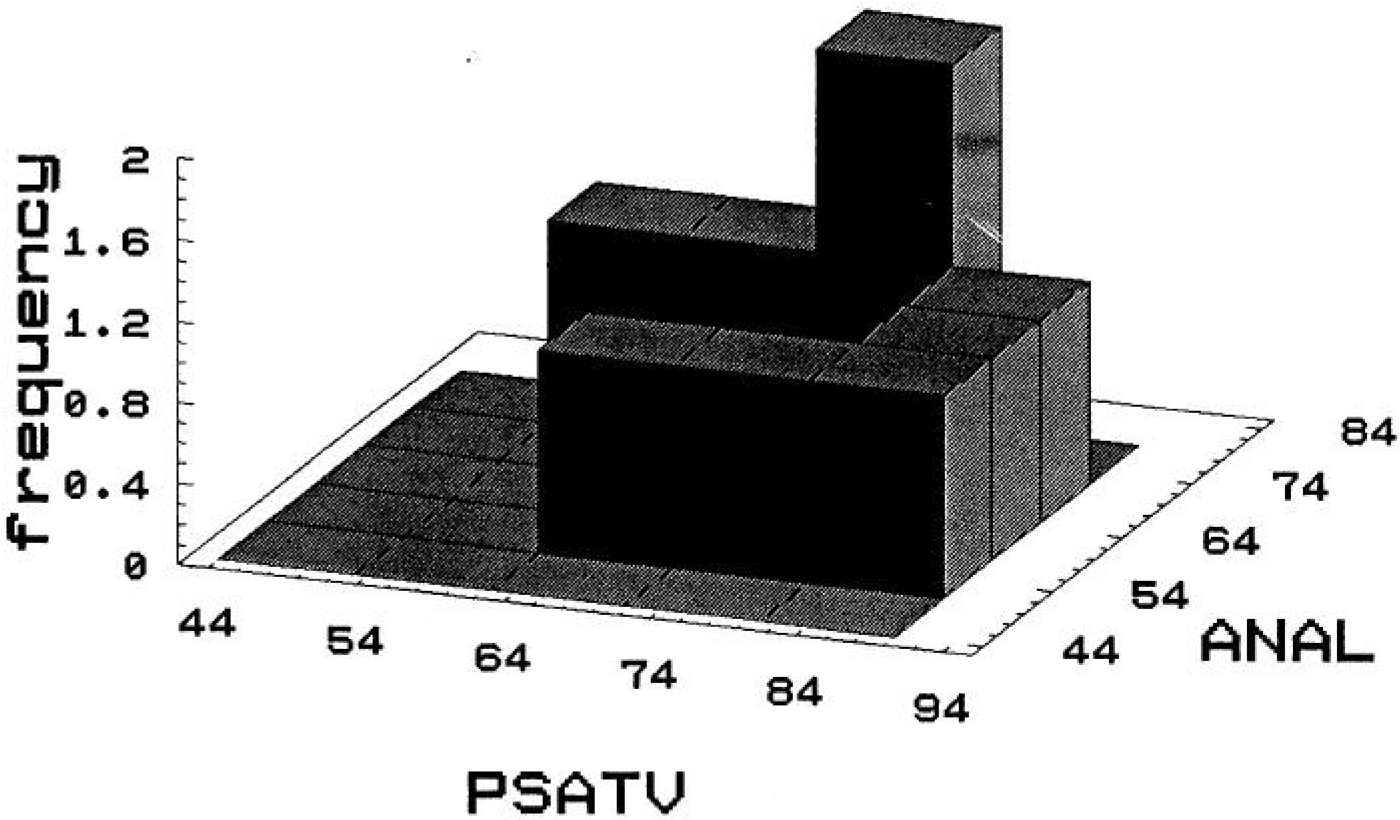
rameter	Estimate	Standard Error	T Value	Prob. Level
tercept	60.0706	28.6266	2.09842	.06224
.ope	0.16183	0.444644	0.363955	.72347

Analysis of Variance

ource	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
odel	38.731354	1	38.731354	.13246	.72347
rror	2923.9353	10	292.3935		
otal (Corr.)	2962.6667	11			

orrelation Coefficient = 0.114338   R-squared = 1.31 percent
tnd. Error of Est. = 17.0995

Three-D Histogram of PSAT verbal scores
vs High school analytic geometry scores



egression Analysis - Linear model: Y = a+bX

ependent variable: ANAL Independent variable: PSATV

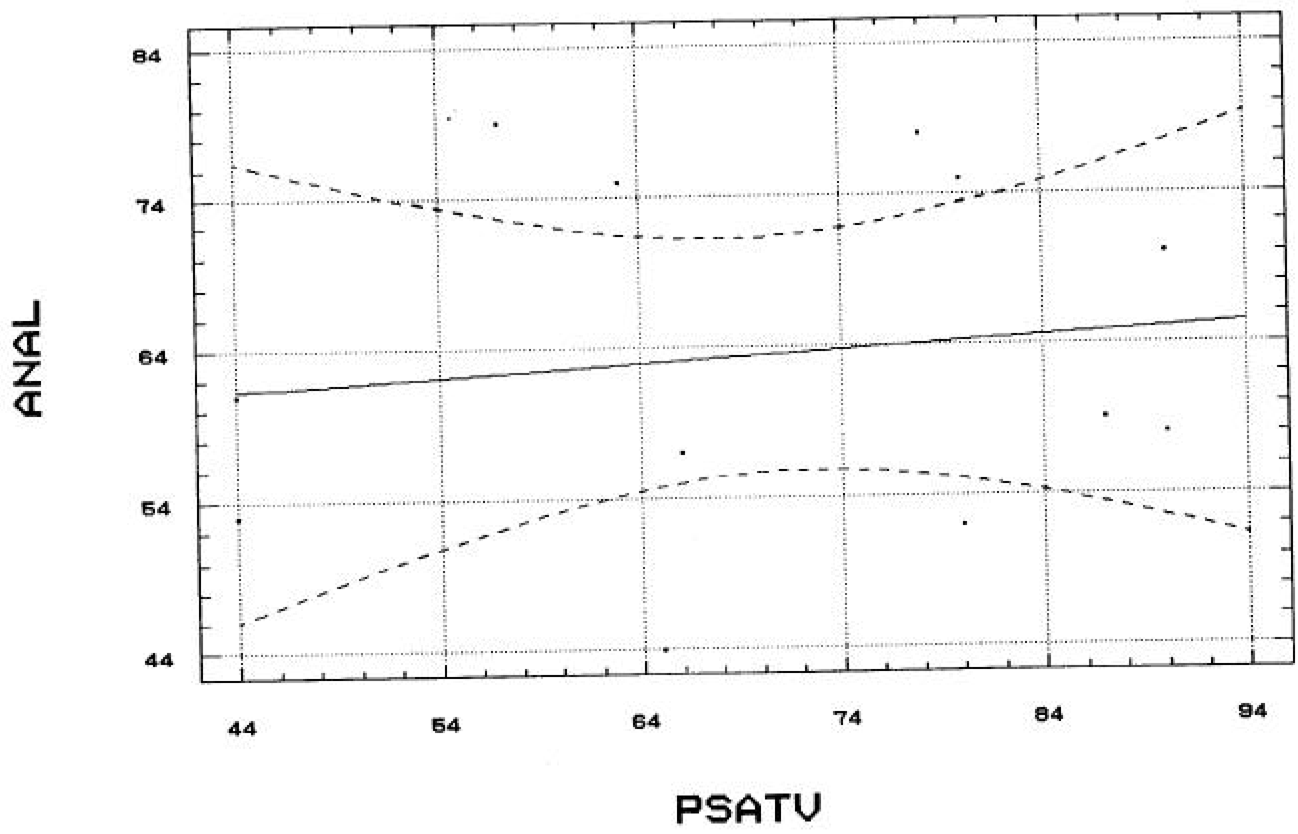
arameter	Estimate	Standard Error	T Value	Prob. Level
ntercept	57.7349	15.9959	3.60935	.00477
lope	0.0807831	0.221959	0.363955	.72347

Analysis of Variance

ource	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
lodel	19.334083	1	19.334083	.13246	.72347
rror	1459.5826	10	145.9583		
Total (Corr.)	1478.9167	11			

Correlation Coefficient = 0.114338 R-squared = 1.31 percent
Stnd. Error of Est. = 12.0813

Regression of ANALytic geometry scores on PSAT verbal aptitude scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: SATV — Independent variable: PSATM

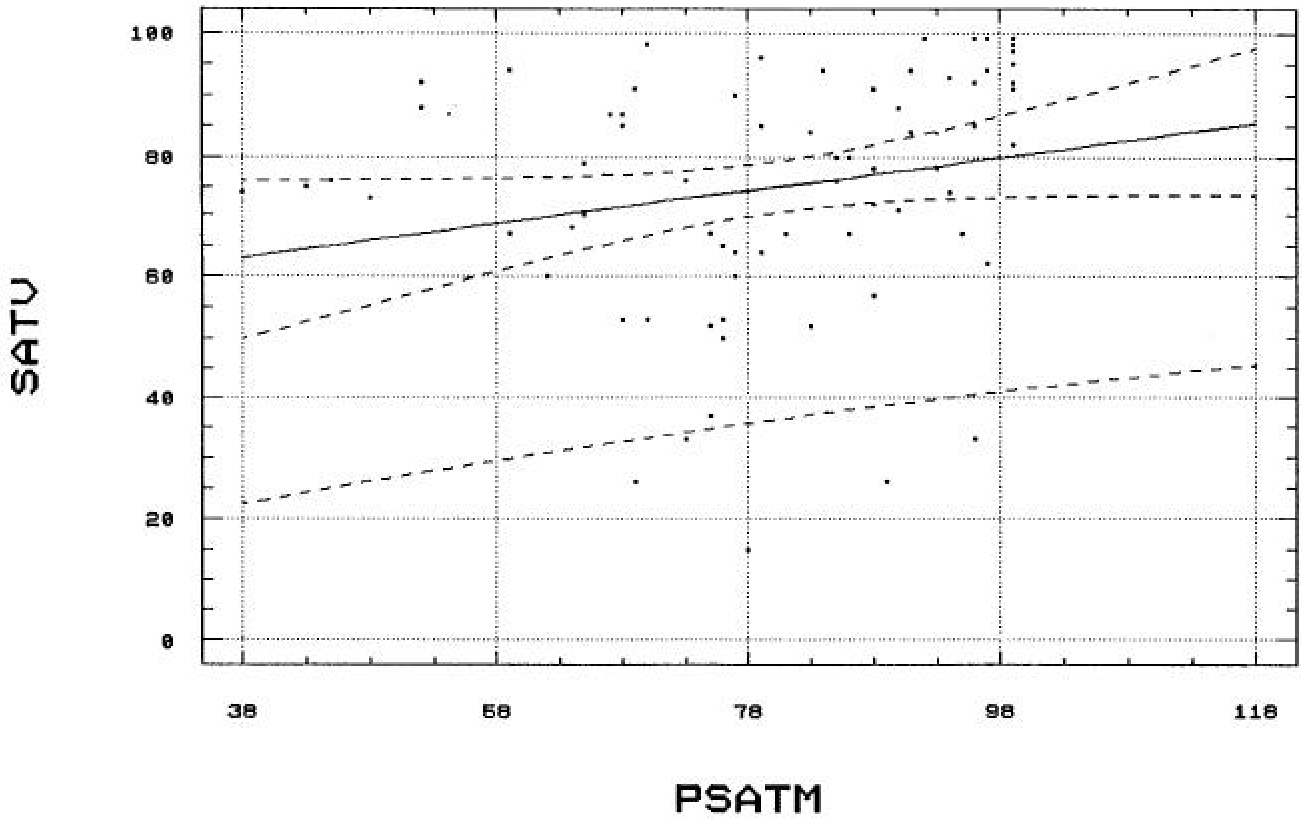
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	52.1958	11.9976	4.35051	.00004
Slope	0.282875	0.147053	1.92363	.05825

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	1367.3537	1	1367.3537	3.700	.05825
Error	27344.581	74	369.521		
Total (Corr.)	28711.934	75			

Correlation Coefficient = 0.218227 R-squared = 4.76 percent
Stnd. Error of Est. = 19.2229

Regression of SAT verbal aptitude scores on PSAT scores in mathematics



Regression Analysis - Linear model: Y = a+bX

Dependent variable: PSATM — Independent variable: SATV

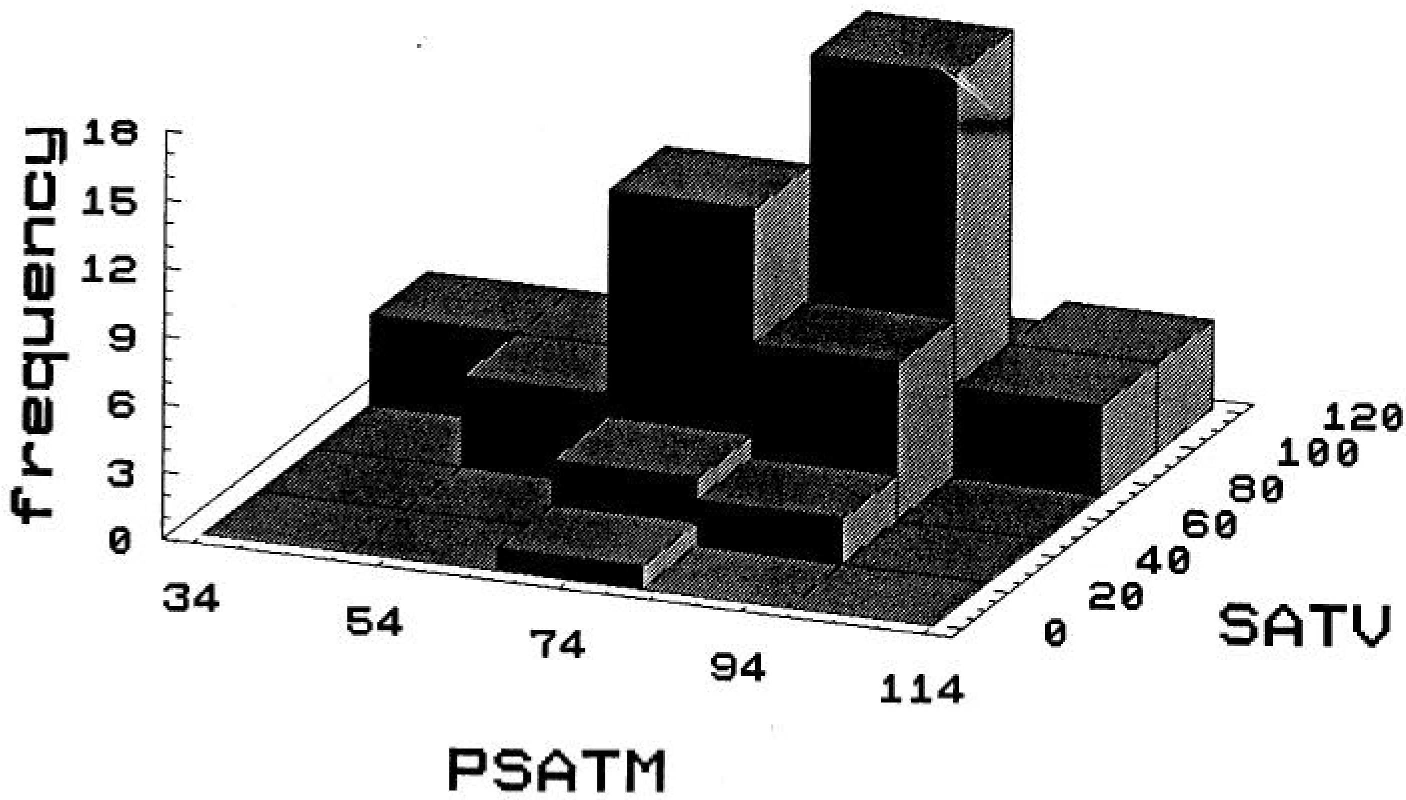
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	67.5907	6.77075	9.98275	.00000
Slope	0.168354	0.0875192	1.92363	.05825

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	813.78681	1	813.78681	3.7003	.05825
Error	16274.253	74	219.922		
Total (Corr.)	17088.039	75			

Correlation Coefficient = 0.218227 R-squared = 4.76 percent
Stnd. Error of Est. = 14.8298

Three-D Histogram of PSAT mathematical aptitude scores vs SAT verbal scores



egression Analysis - Linear model: Y = a+bX

ependent variable: PSATM   Independent variable: SATM

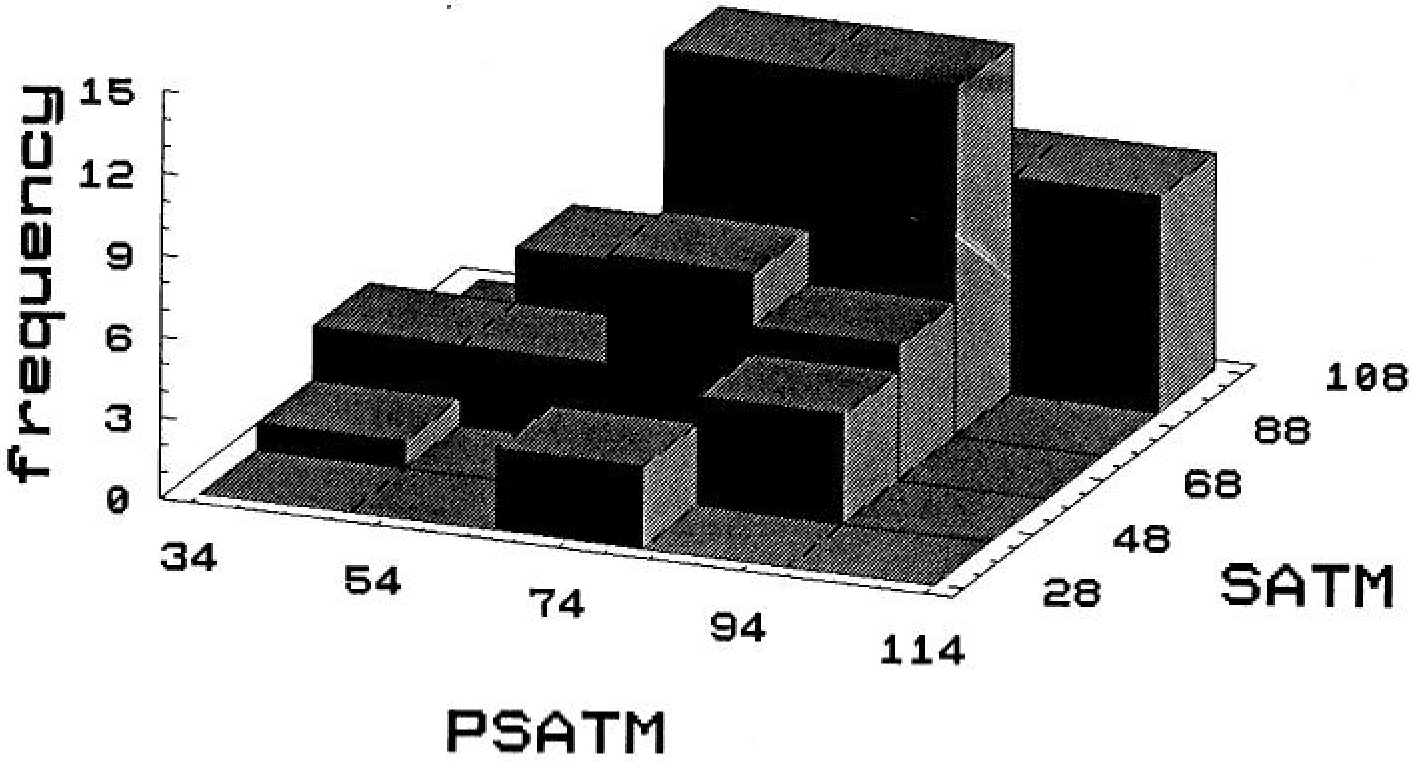
arameter	Estimate	Standard Error	T Value	Prob. Level
ntercept	50.4183	8.0961	6.22748	.00000
lope	0.372116	0.0991783	3.75199	.00035

Analysis of Variance

ource	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
odel	2731.1879	1	2731.1879	14.077	.00035
rror	14356.852	74	194.012		
otal (Corr.)	17088.039	75			

orrelation Coefficient = 0.399788   R-squared = 15.98 percent
tnd. Error of Est. = 13.9288

Three-D Histogram of PSAT mathematical scores vs SAT math aptitude scores



Regression Analysis - Linear model: Y = a+bX

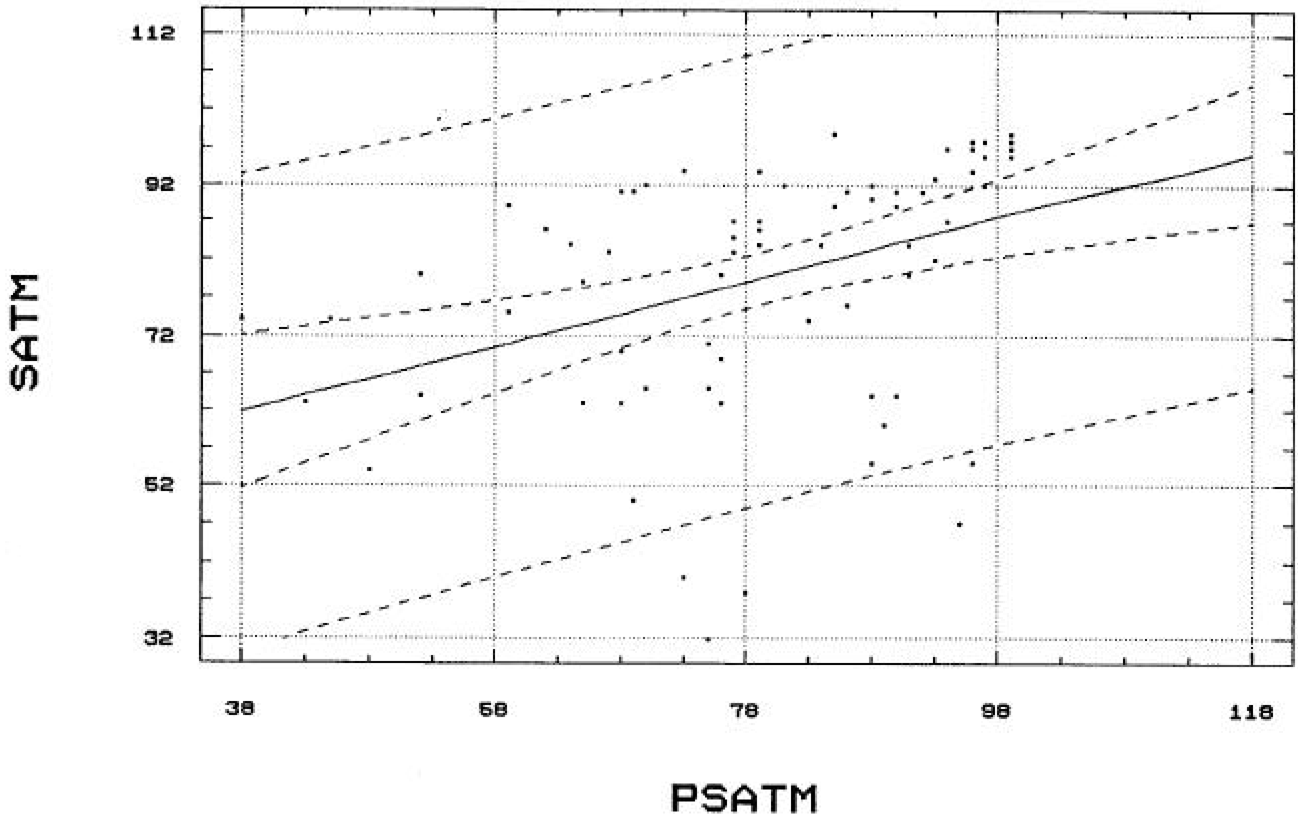
Dependent variable: SATM				Independent variable: PSAT
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	45.5802	9.33985	4.88018	.00001
Slope	0.429517	0.114477	3.75199	.00035

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	3152.4860	1	3152.4860	14.077	.00035
Error	16571.461	74	223.939		
Total (Corr.)	19723.947	75			

Correlation Coefficient = 0.399788 R-squared = 15.98 percent
Stnd. Error of Est. = 14.9646

Regression of SAT scores in mathematics on PSAT scores in mathematics



Regression Analysis - Linear model: Y = a+bX

Dependent variable: GEOM | Independent variable: PSATM

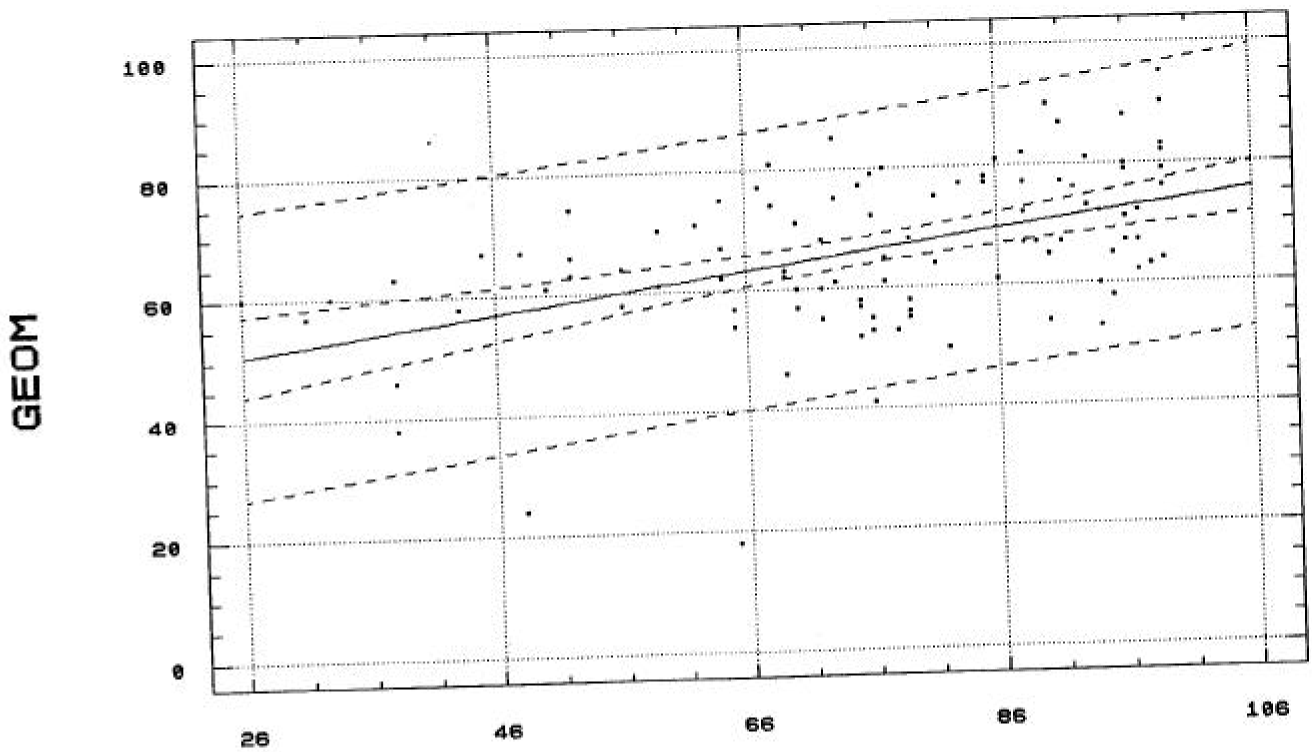
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	42.6997	4.97702	8.57936	.00000
Slope	0.310817	0.0636748	4.88133	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	3183.2679	1	3183.2679	23.827	.00000
Error	13359.722	100	133.597		
Total (Corr.)	16542.990	101			

Correlation Coefficient = 0.438662 R-squared = 19.24 percent
Stnd. Error of Est. = 11.5584

Regression of high school GEOMetry scores on PSAT scores in mathematics



Regression Analysis - Linear model: Y = a+bX

Dependent variable: PSATM   Independent variable: GEOM

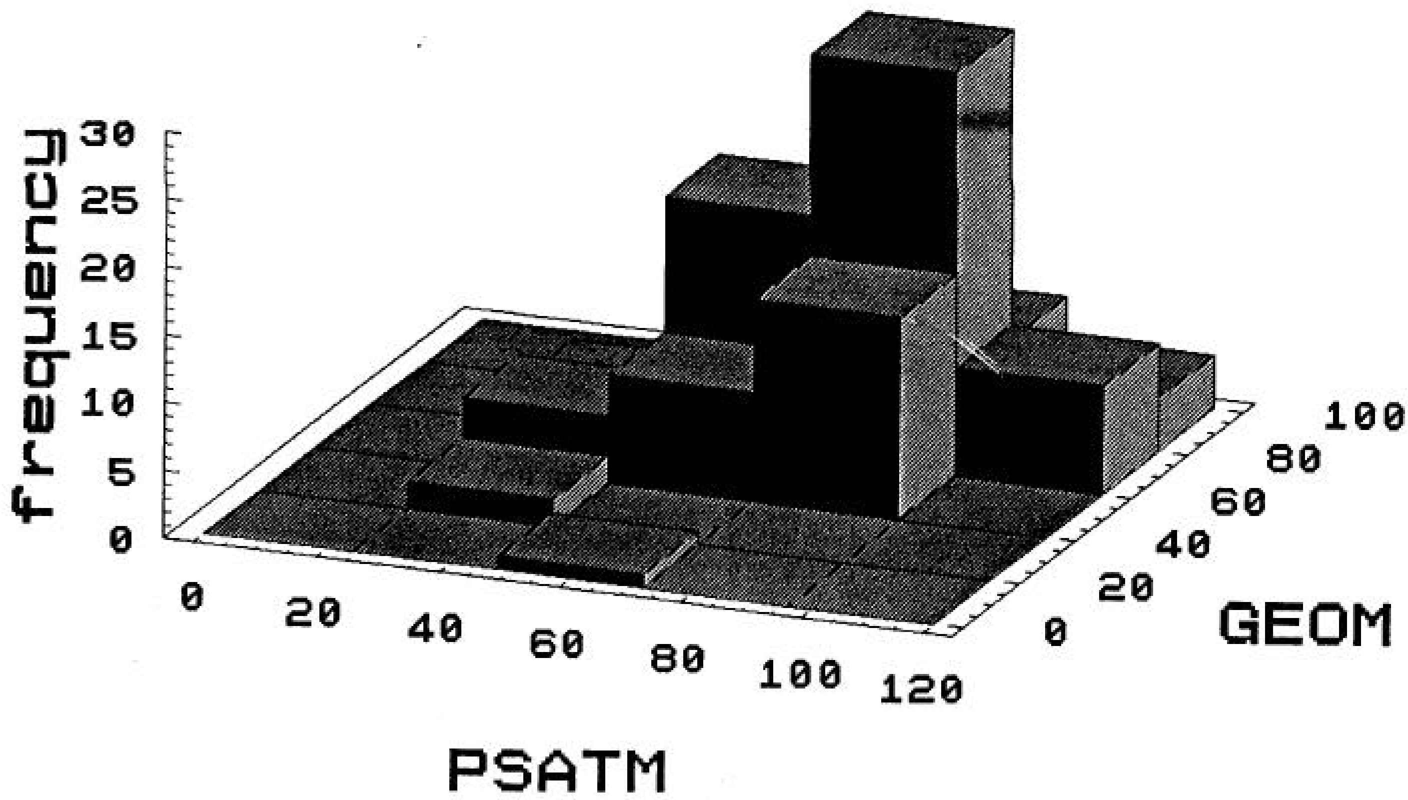
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	34.9963	8.5678	4.08462	.00009
Slope	0.61909	0.126828	4.88133	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	6340.4698	1	6340.4698	23.827	.00000
Error	26610.050	100	266.100		
Total (Corr.)	32950.520	101			

Correlation Coefficient = 0.438662   R-squared = 19.24 percent
Stnd. Error of Est. = 16.3126

Three-D Histogram of PSAT mathematical scores vs elementary geometry scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: PSATM

Independent variable: EA

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	20.3259	5.97021	3.40455	.00088
Slope	0.783181	0.0820575	9.5443	.00000

Analysis of Variance

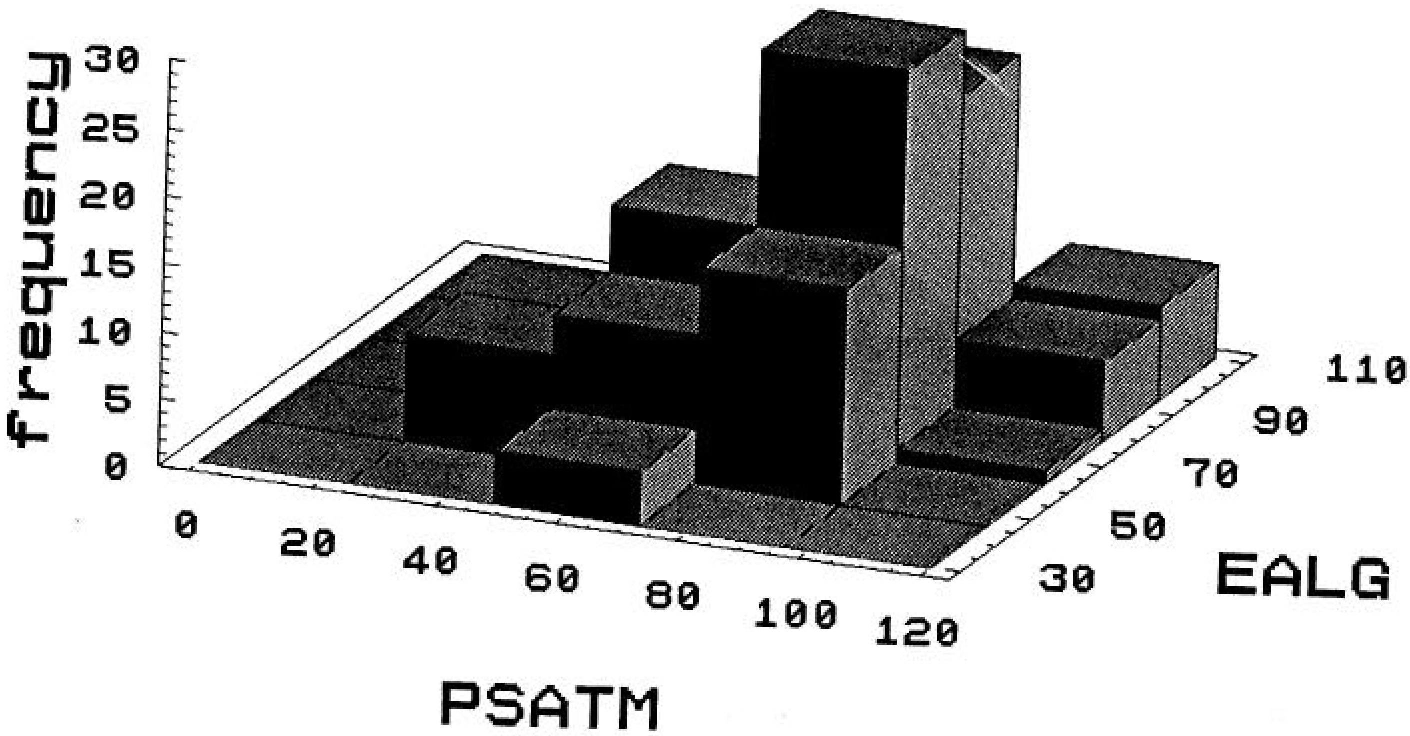
Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	15834.300	1	15834.300	91.09	.00000
Error	22423.364	129	173.825		
Total (Corr.)	38257.664	130			

Correlation Coefficient = 0.643339

R-squared = 41.39 percent

Stnd. Error of Est. = 13.1843

Three-D Histogram of PSAT mathematical scores vs elementary algebra scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: EALG | Independent variable: PSATM

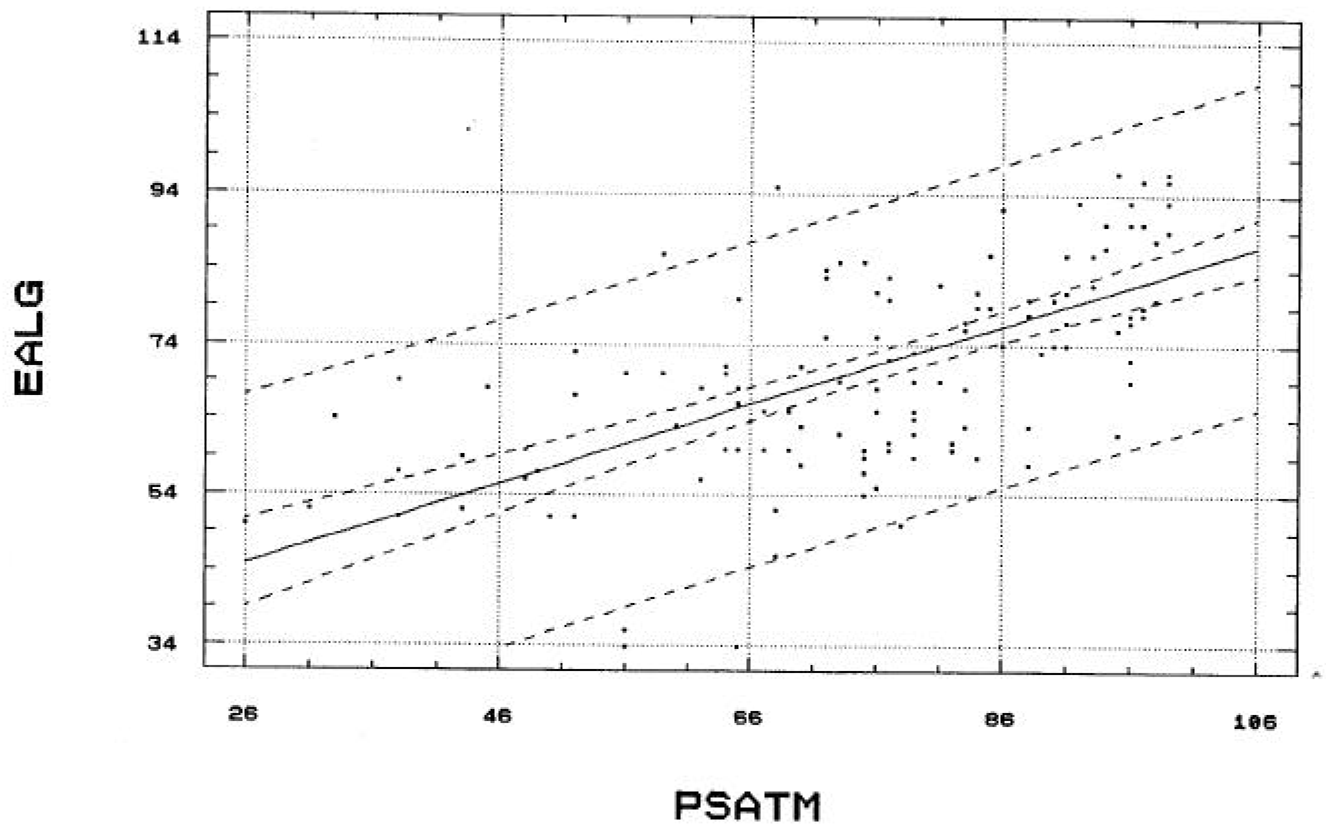
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	31.1007	4.32598	7.1893	.00000
Slope	0.528467	0.05537	9.5443	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	10684.519	1	10684.519	91.09	.00000
Error	15130.626	129	117.292		
Total (Corr.)	25815.145	130			

Correlation Coefficient = 0.643339 R-squared = 41.39 percent
Stnd. Error of Est. = 10.8301

Regression of high school elementary algebra scores on PSAT math score



Regression Analysis - Linear model: Y = a+bX

Dependent variable: TRIG　　　Independent variable: PSA[illegible]

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	41.6305	9.19099	4.52949	.00002
Slope	0.303789	0.113079	2.68651	.00886

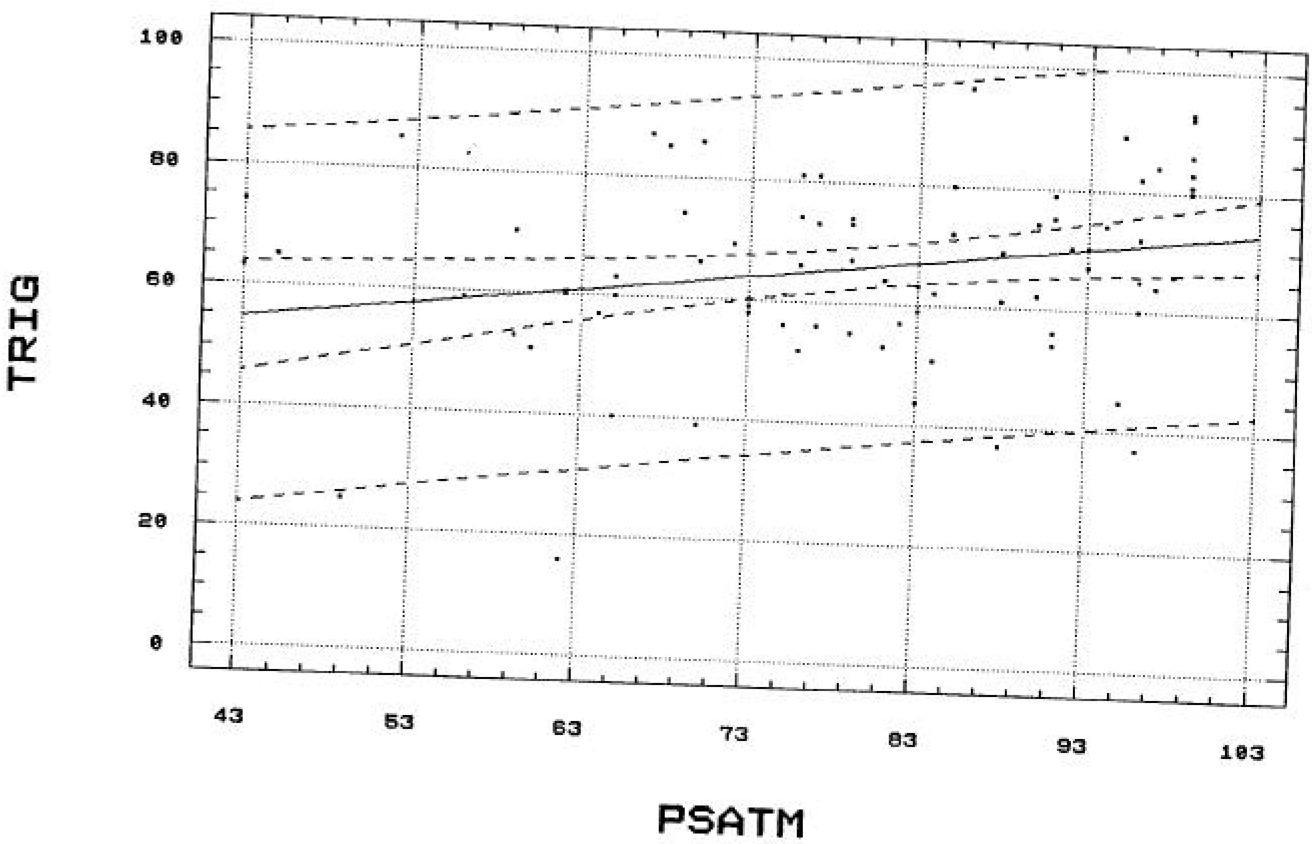
Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	1573.4632	1	1573.4632	7.217	.00886
Error	16568.909	76	218.012		
Total (Corr.)	18142.372	77			

Correlation Coefficient = 0.294497　　R-squared = 8.67 percent

Stnd. Error of Est. = 14.7652

Regression of high school trigonometry scores on PSAT scores in mathematics



Regression Analysis - Linear model: Y = a+bX

Dependent variable: PSATM   Independent variable: TRI(

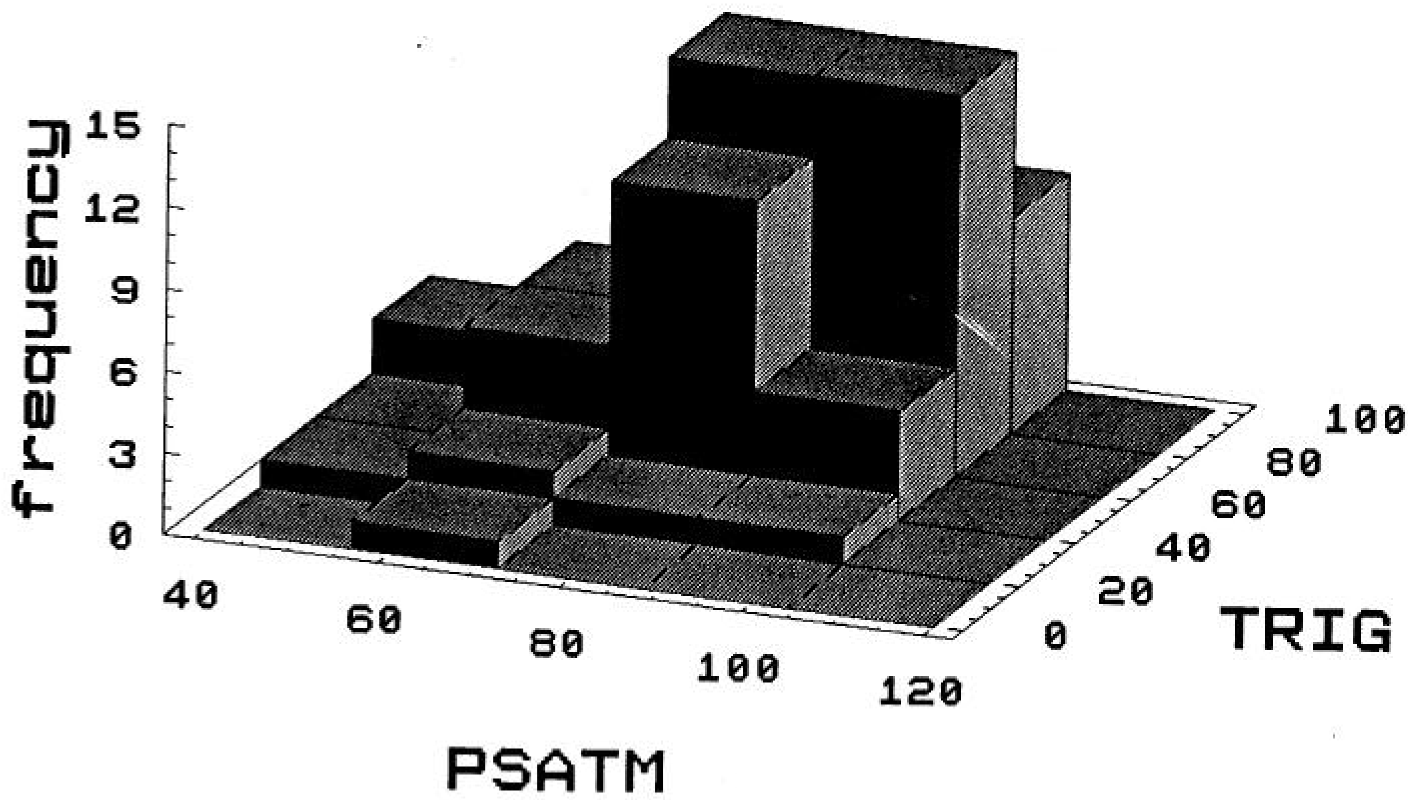
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	61.1064	7.18921	8.49973	.00000
Slope	0.28549	0.106268	2.68651	.00886

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	1478.6833	1	1478.6833	7.217	.00886
Error	15570.855	76	204.880		
Total (Corr.)	17049.538	77			

Correlation Coefficient = 0.294497   R-squared =   8.67 percent
Stnd. Error of Est. = 14.3136

Three-D Histogram of PSAT mathematical scores vs elementary trigonometry scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: PSATM — Independent variable: IALG

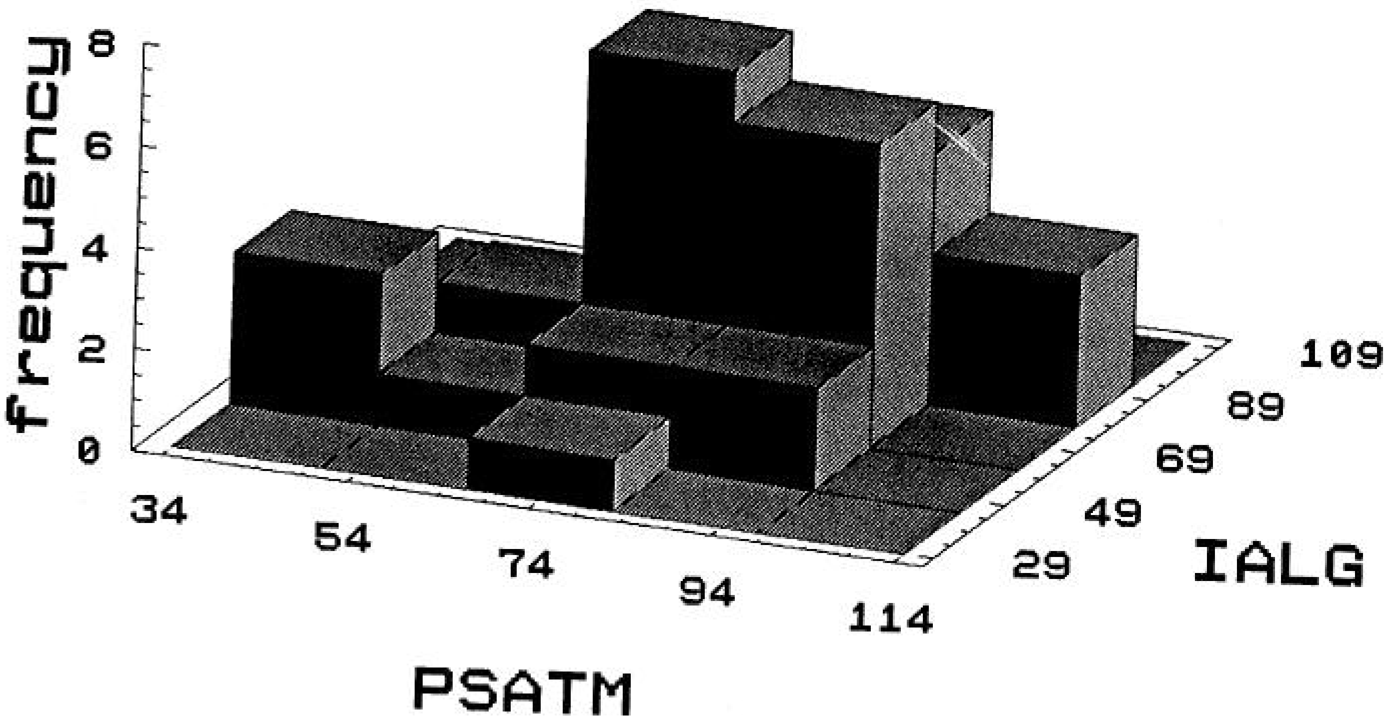
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	36.8808	13.5756	2.71669	.01055
Slope	0.601291	0.19161	3.13809	.00364

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	2228.7385	1	2228.7385	9.848	.00364
Error	7242.3203	32	226.3225		
Total (Corr.)	9471.0588	33			

Correlation Coefficient = 0.485099 R-squared = 23.53 percent
Stnd. Error of Est. = 15.044

Three-D Histogram of PSAT mathematical scores vs intermediate algebra scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: IALG   Independent variable: PSATM

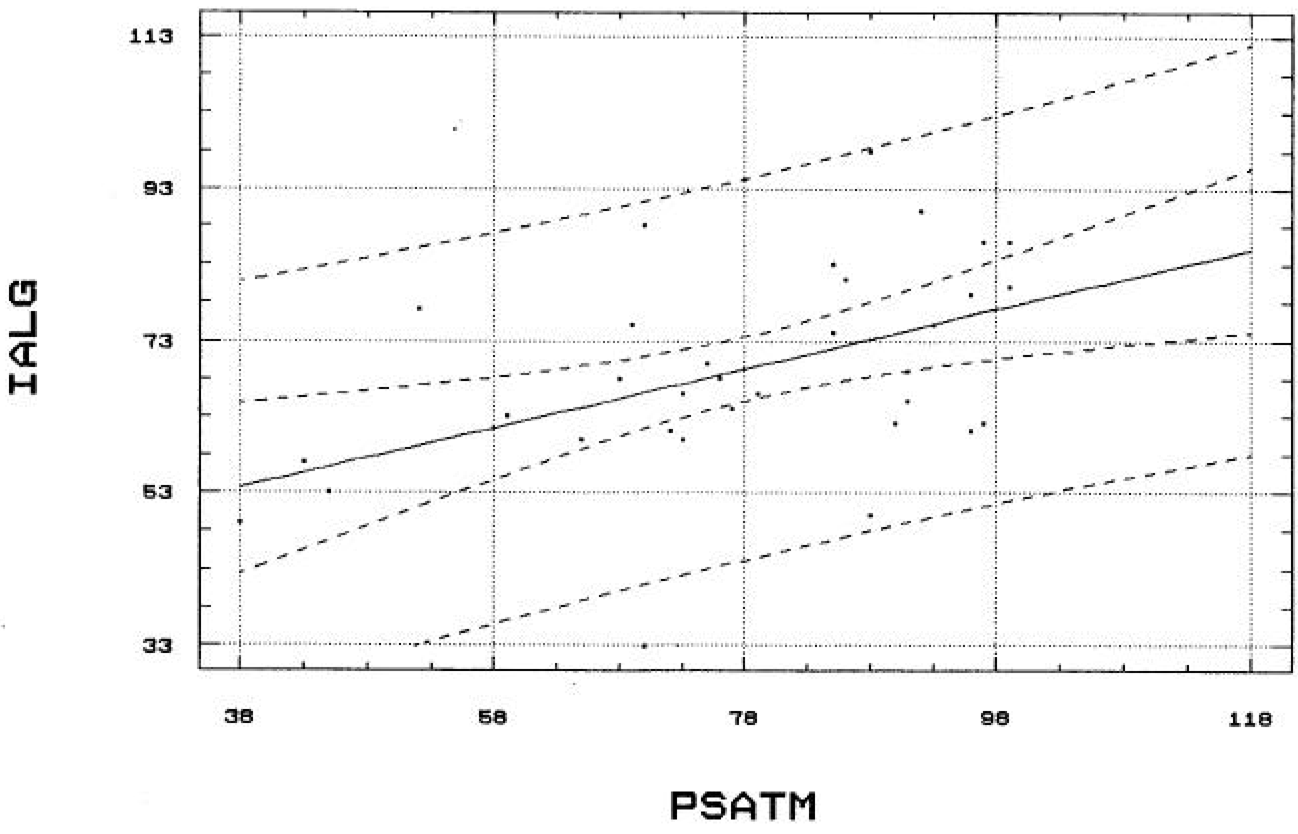
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	38.7565	10.0339	3.86257	.00051
Slope	0.391359	0.124713	3.13809	.00364

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	1450.6083	1	1450.6083	9.848	.00364
Error	4713.7741	32	147.3054		
Total (Corr.)	6164.3824	33			

Correlation Coefficient = 0.485099   R-squared = 23.53 percent
Stnd. Error of Est. = 12.1369

Regression of intermediate algebra scores on PSAT scores in mathematics



Regression Analysis - Linear model: Y = a+bX

Dependent variable: ANAL Independent variable: PSAT

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	74.0001	17.0468	4.34099	.00146
Slope	-0.140488	0.221624	-0.633902	.54037

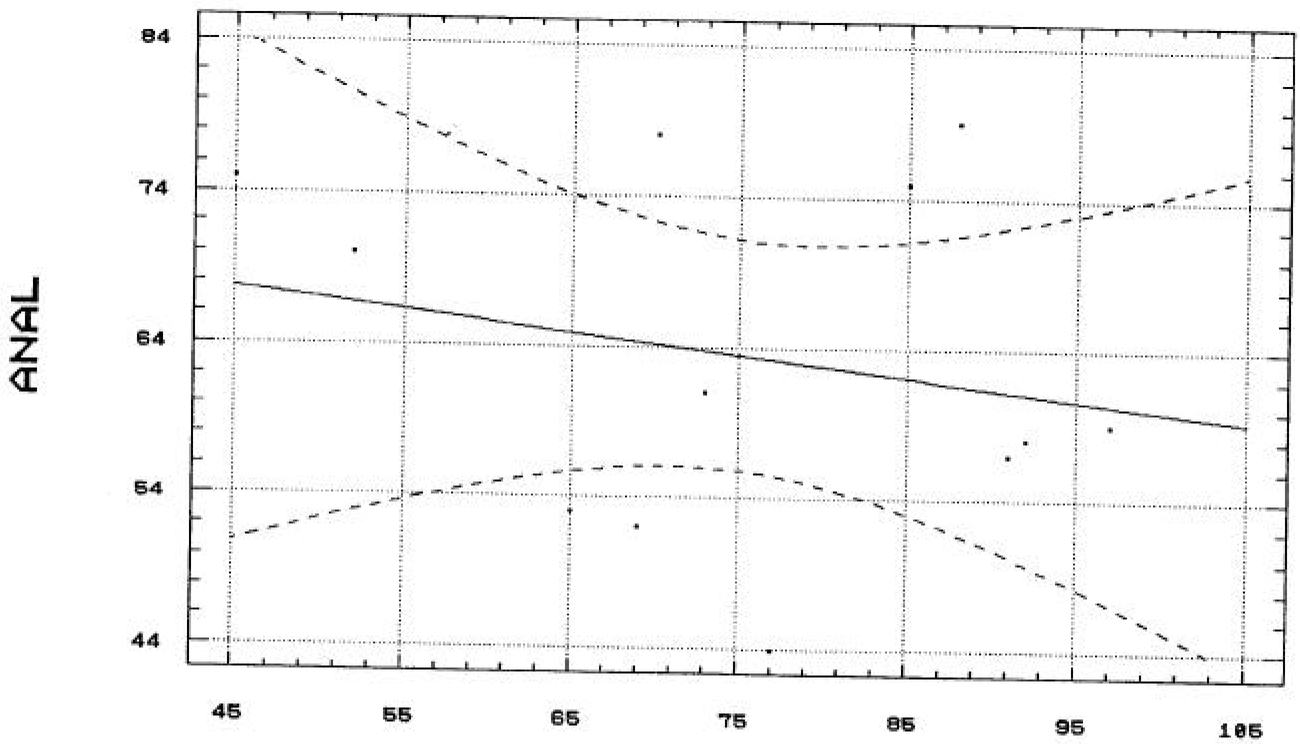
Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	57.131890	1	57.131890	.40183	.54037
Error	1421.7848	10	142.1785		
Total (Corr.)	1478.9167	11			

Correlation Coefficient = -0.196547
Stnd. Error of Est. = 11.9239

R-squared = 3.86 percent

Regression of ANALytic geometry scores on PSAT scores in mathematics



Regression Analysis - Linear model: Y = a+bX

Dependent variable: PSATM — Independent variable: ANA

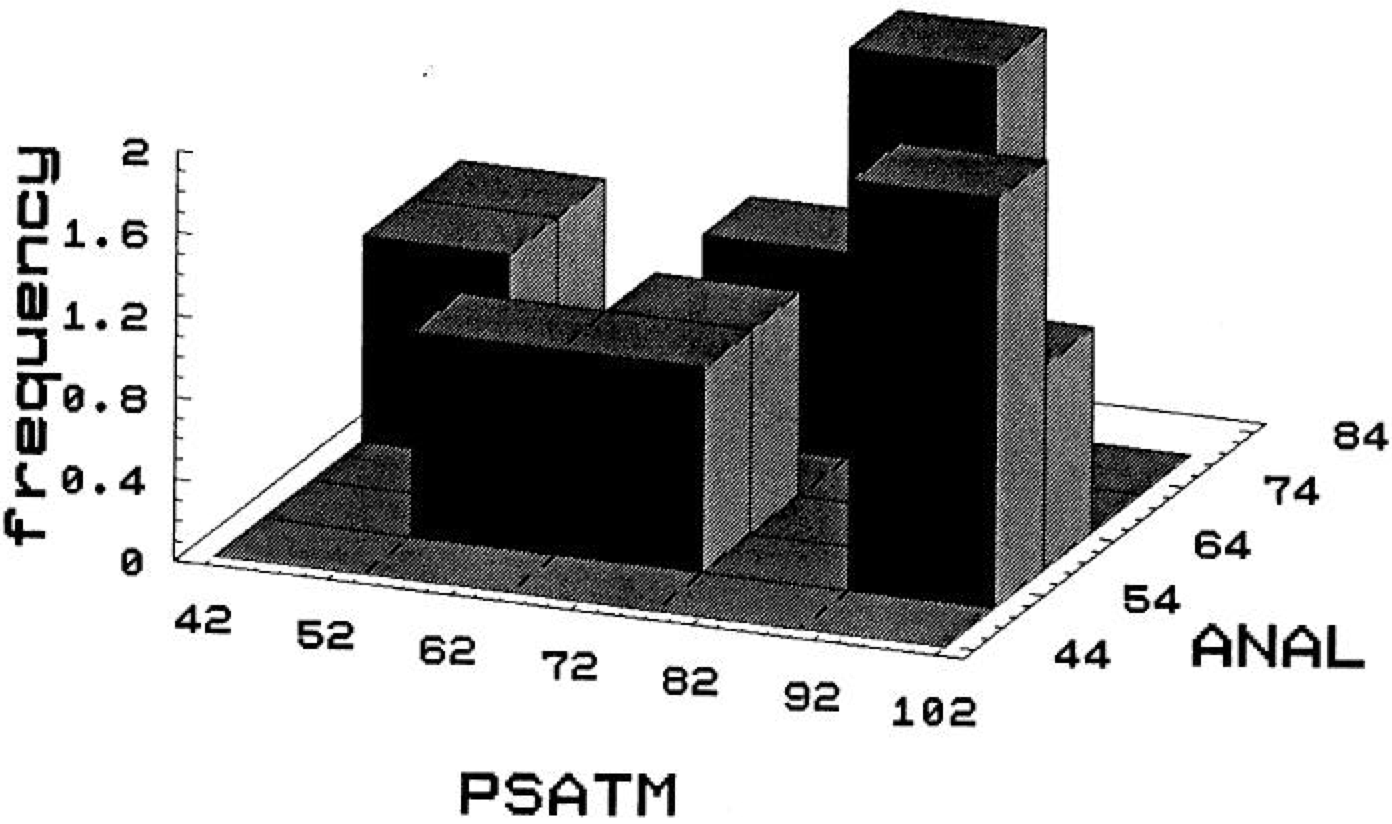
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	92.7714	27.9274	3.32188	.00772
Slope	-0.274976	0.433783	-0.633902	.54037

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	111.82359	1	111.82359	.4018	.54037
Error	2782.8431	10	278.2843		
Total (Corr.)	2894.6667	11			

Correlation Coefficient = -0.196547 R-squared = 3.86 percent
Stnd. Error of Est. = 16.6819

Three-D Histogram of PSAT mathematical scores vs analytic geometry scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: SATV Independent variable: SAT

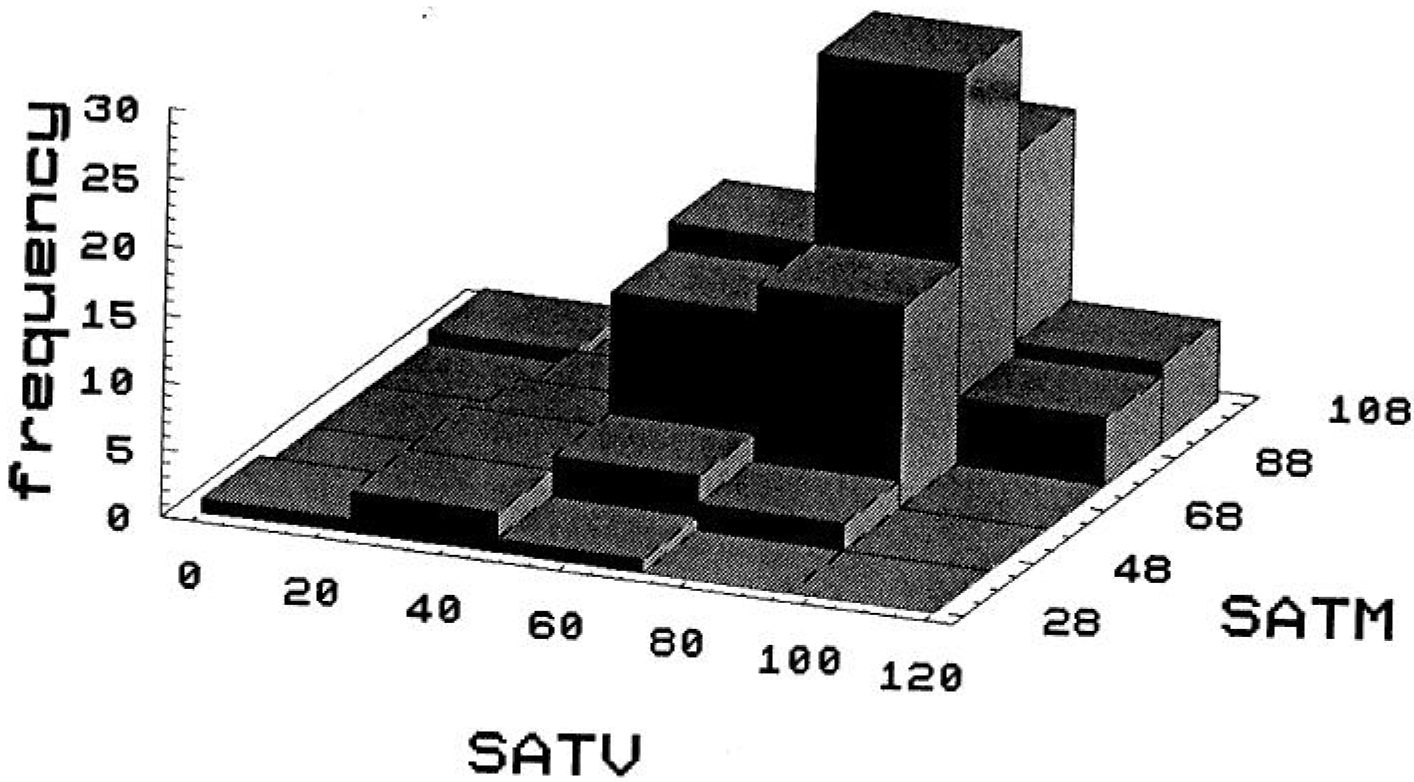
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	18.8948	7.37314	2.56265	.01161
Slope	0.703138	0.0902566	7.79043	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	15042.089	1	15042.089	60.69	.00000
Error	29989.586	121	247.848		
Total (Corr.)	45031.675	122			

Correlation Coefficient = 0.577956 R-squared = 33.40 percent
Stnd. Error of Est. = 15.7432

Three-D Histogram of SAT verbal aptitude scores vs SAT mathematical scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: SATM Independent variable: SAT

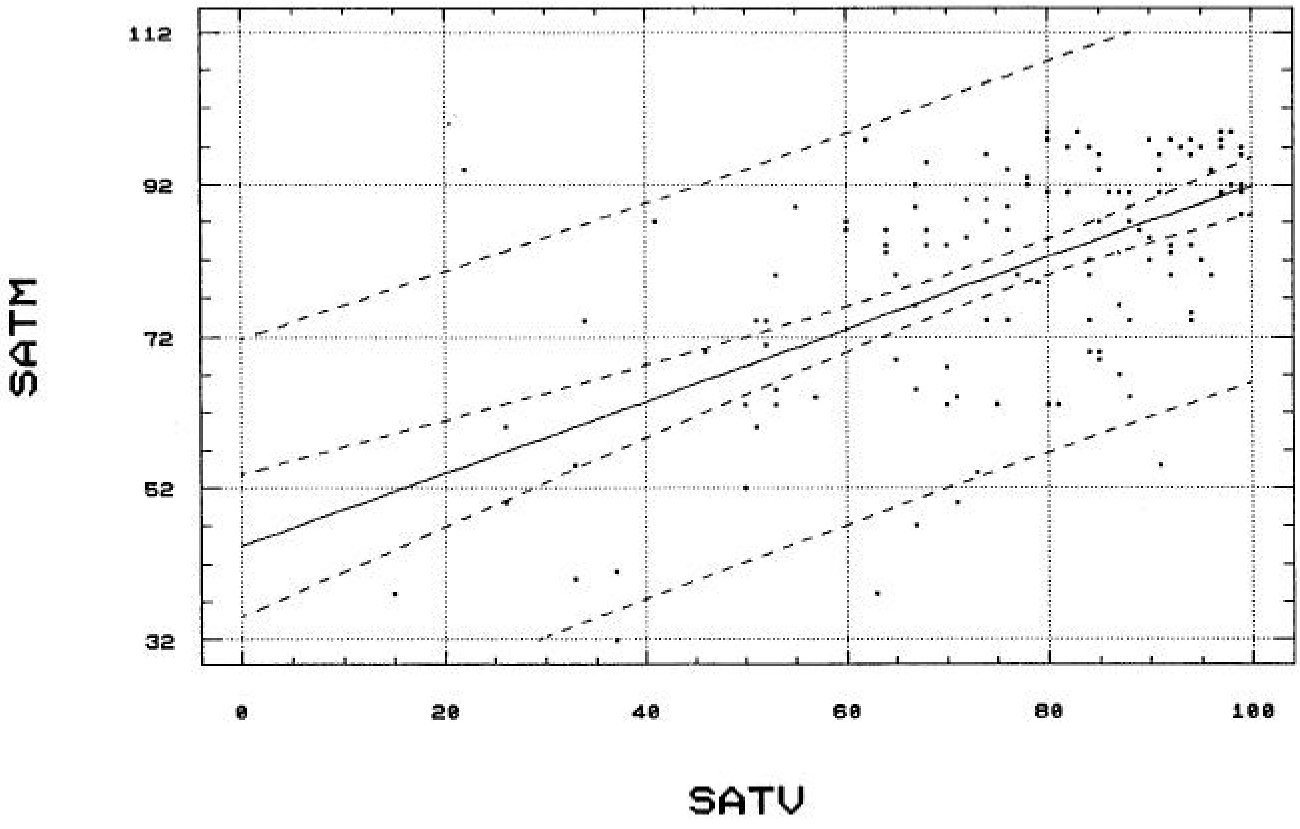
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	44.4094	4.73537	9.37824	.00000
Slope	0.475061	0.0609801	7.79043	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	10162.885	1	10162.885	60.69	.00000
Error	20261.863	121	167.453		
Total (Corr.)	30424.748	122			

Correlation Coefficient = 0.577956 R-squared = 33.40 percent
Stnd. Error of Est. = 12.9404

Regression of SAT scores in mathematics on SAT verbal aptitude scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: GEOM

Independent variable: SA[illegible]

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	57.7295	4.79399	12.042	.00000
Slope	0.14503	0.0617944	2.34698	.02086

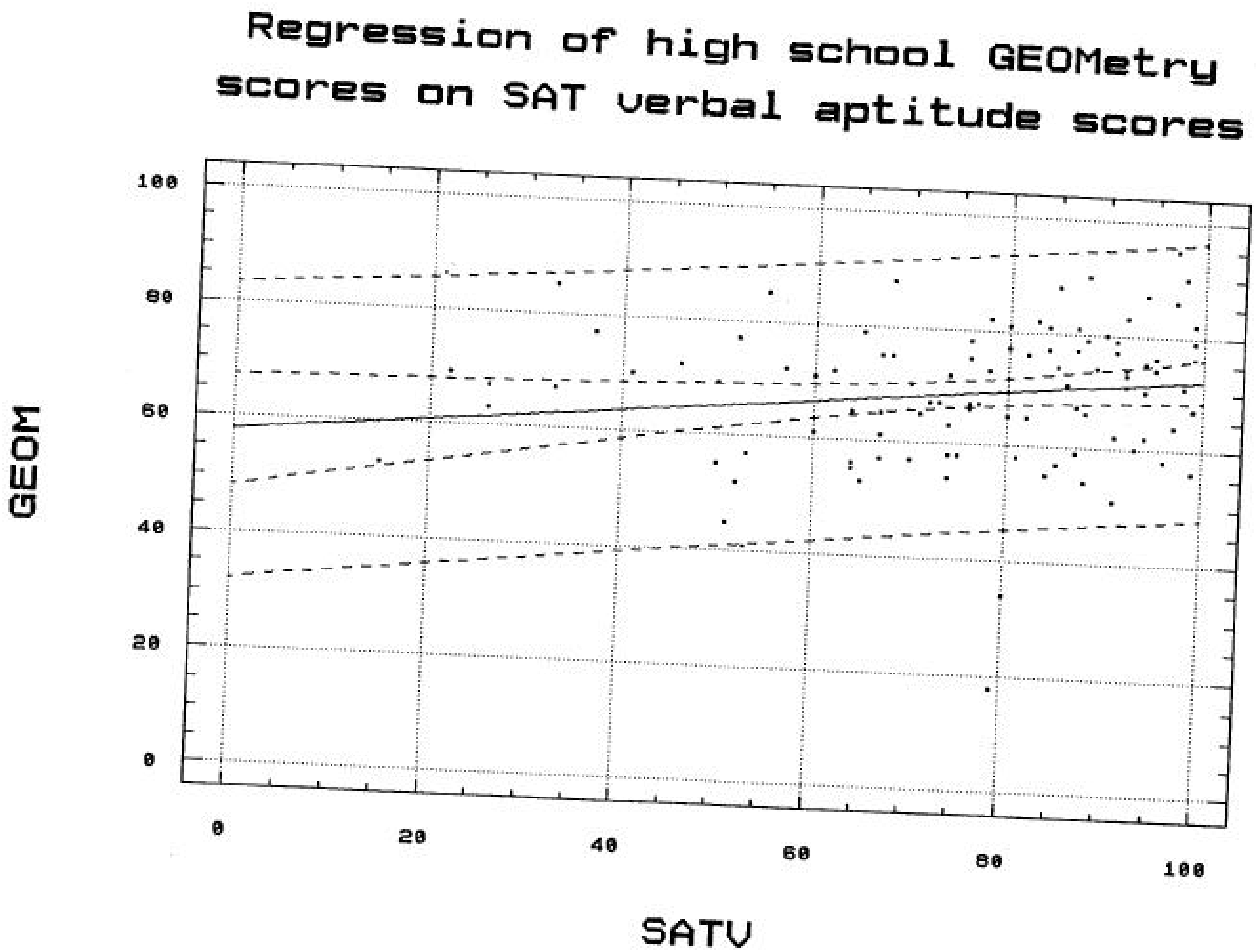
Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	797.85573	1	797.85573	5.5083	.02086
Error	14774.260	102	144.846		
Total (Corr.)	15572.115	103			

Correlation Coefficient = 0.226354

Stnd. Error of Est. = 12.0352

R-squared = 5.12 percent



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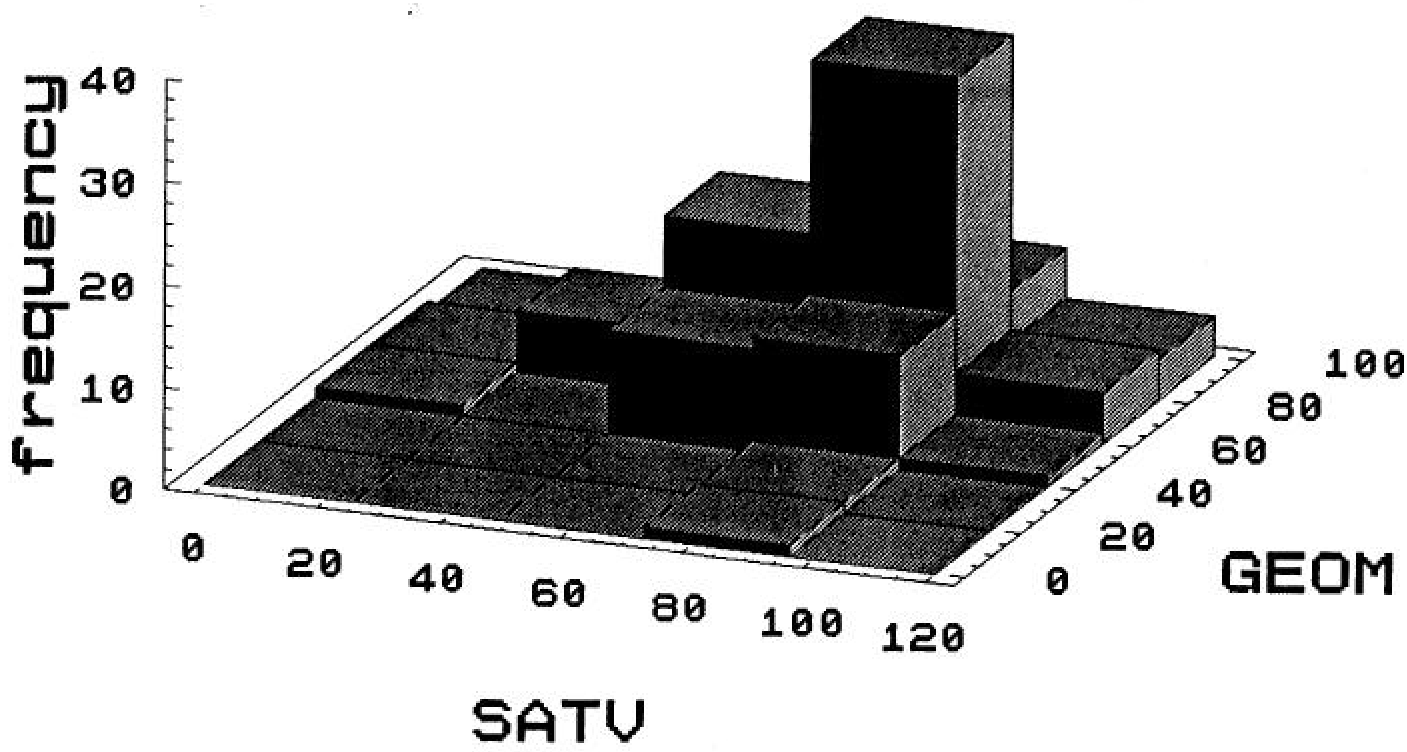
Regression Analysis - Linear model: Y = a+bX
------------------------------------------------------------------------------
Dependent variable: SATV                          Independent variable: GEOM
------------------------------------------------------------------------------
                                 Standard              T              Prob.
Parameter        Estimate          Error             Value            Level
------------------------------------------------------------------------------
Intercept         50.9451         10.4941           4.85463          .00000
Slope            0.353279        0.150525           2.34698          .02086
------------------------------------------------------------------------------

                          Analysis of Variance
------------------------------------------------------------------------------
Source             Sum of Squares     Df  Mean Square     F-Ratio  Prob. Level
Model                   1943.4987      1    1943.4987       5.508       .02086
Error                   35988.655    102      352.830
------------------------------------------------------------------------------
Total (Corr.)           37932.154    103

Correlation Coefficient = 0.226354            R-squared =    5.12 percent
Stnd. Error of Est. = 18.7838
```

Three-D Histogram of SAT verbal aptitude scores vs elementary geometry scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: SATV

Independent variable: E/

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	30.3466	9.44403	3.21331	.00173
Slope	0.609686	0.12751	4.78149	.00001

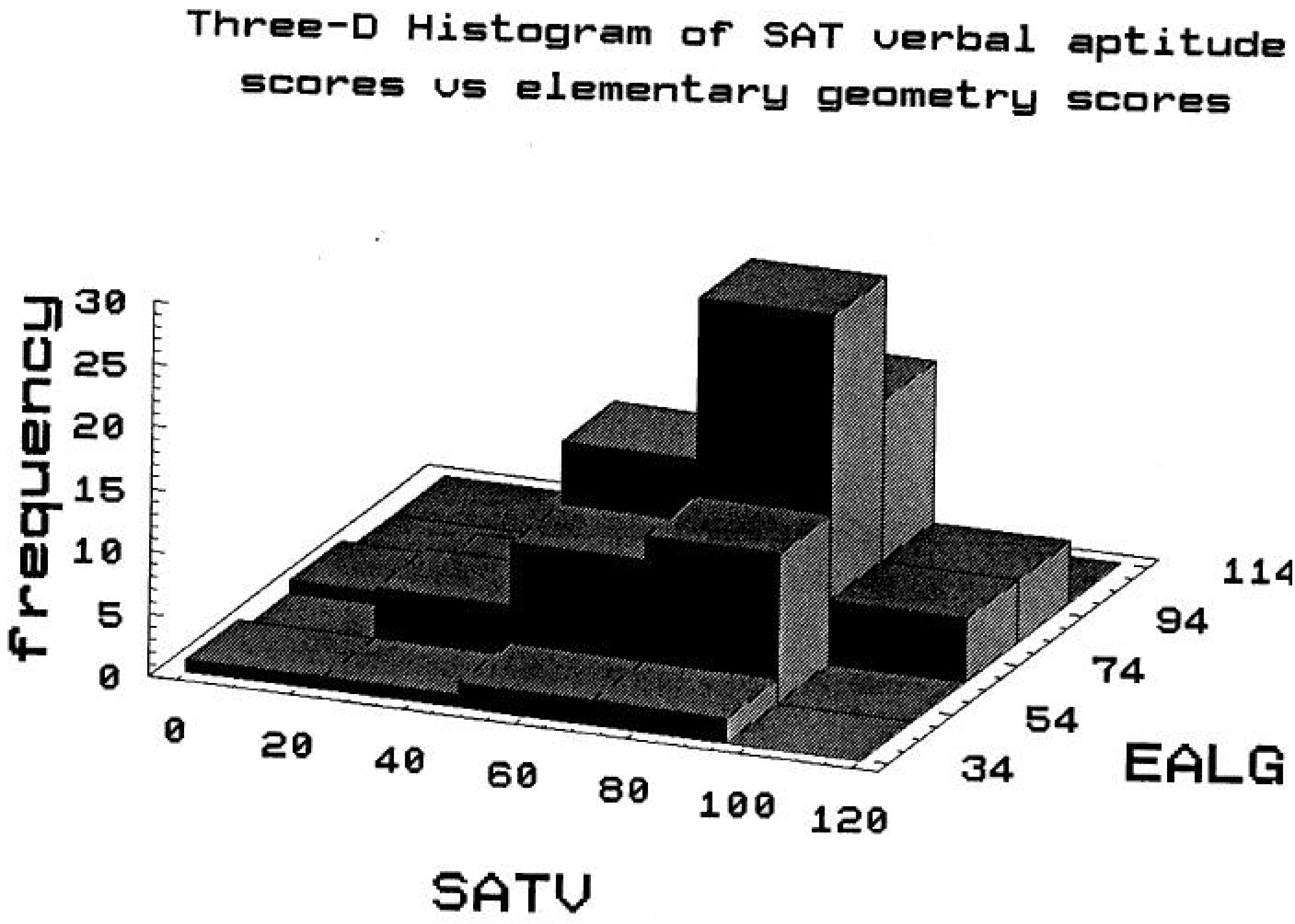
Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	7331.6233	1	7331.6233	22.863	.00001
Error	34954.287	109	320.682		
Total (Corr.)	42285.910	110			

Correlation Coefficient = 0.416392

R-squared = 17.34 percent

Stnd. Error of Est. = 17.9076



Regression Analysis - Linear model: Y = a+bX

Dependent variable: EALG | Independent variable: SATV

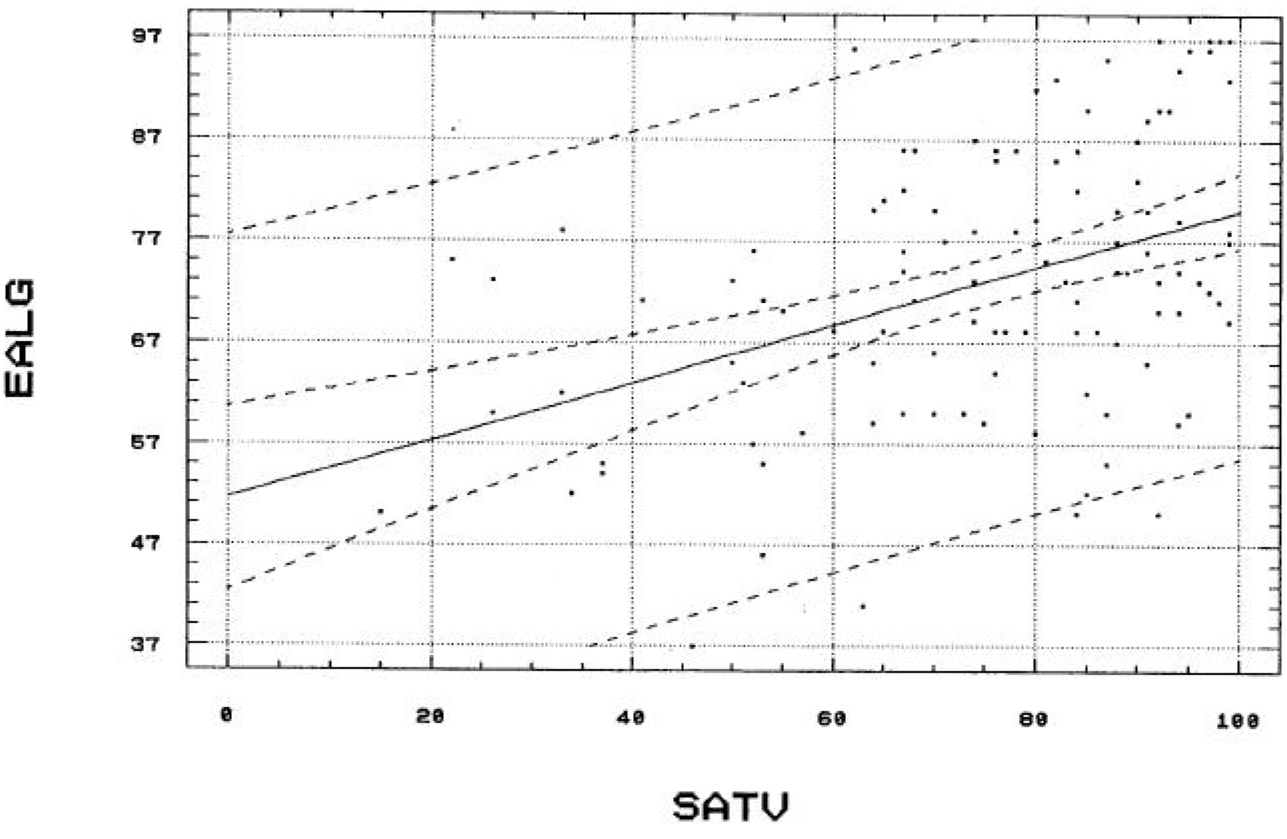
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	51.594	4.59573	11.2265	.00000
Slope	0.28438	0.0594751	4.78149	.00001

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	3419.7370	1	3419.7370	22.863	.00001
Error	16303.957	109	149.578		
Total (Corr.)	19723.694	110			

Correlation Coefficient = 0.416392 R-squared = 17.34 percent
Stnd. Error of Est. = 12.2302

Regression of scores in elementary algebra on SAT verbal aptitude scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: TRIG   Independent variable: SATV

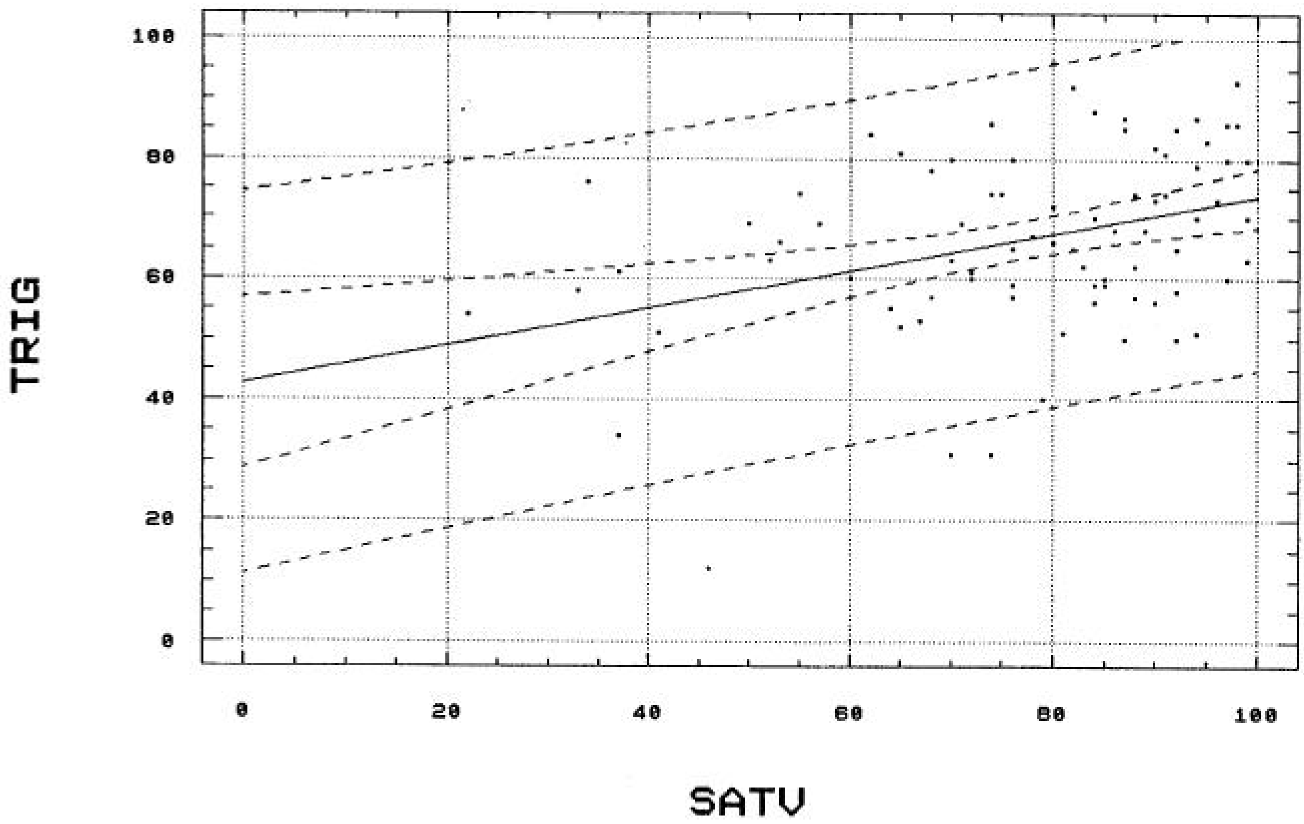
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	42.7804	7.05226	6.06621	.00000
Slope	0.307211	0.0890502	3.44986	.00089

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	2408.0396	1	2408.0396	11.902	.00089
Error	16388.707	81	202.330		
Total (Corr.)	18796.747	82			

Correlation Coefficient = 0.357924   R-squared = 12.81 percent
Stnd. Error of Est. = 14.2243

Regression of high school trigonometry scores on SAT verbal aptitude scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: SATV — Independent variable: TRI(

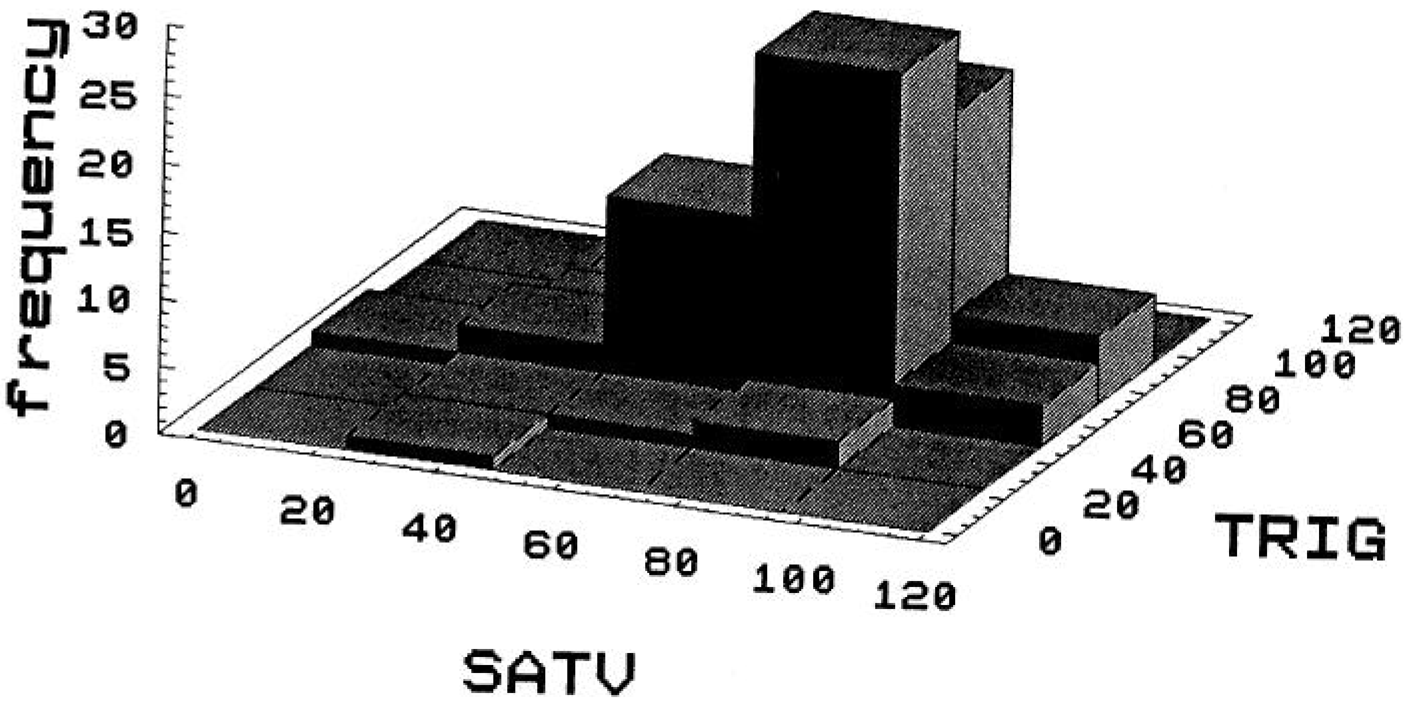
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	49.4954	8.24225	6.00508	.00000
Slope	0.417008	0.120877	3.44986	.00089

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	3268.6660	1	3268.6660	11.902	.00089
Error	22245.985	81	274.642		
Total (Corr.)	25514.651	82			

Correlation Coefficient = 0.357924 R-squared = 12.81 percent
Stnd. Error of Est. = 16.5723

Three-D Histogram of SAT verbal aptitude scores vs elementary trigonometry scores



Regression Analysis - Linear model: Y = a+bX

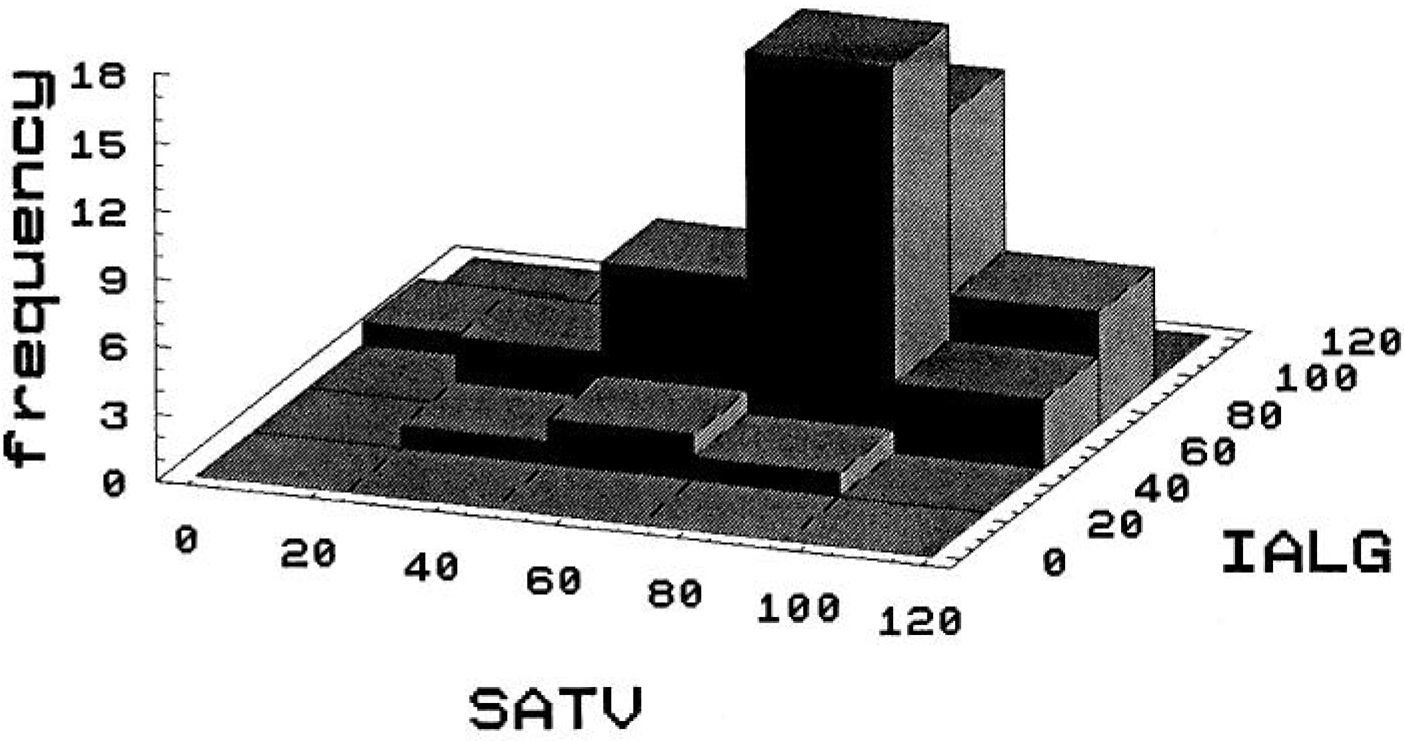
Dependent variable: SATV				Independent variable: IALG
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	56.6183	11.6528	4.85876	.00001
Slope	0.309937	0.166544	1.861	.06800

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	1185.6047	1	1185.6047	3.463	.06800
Error	19170.671	56	342.333		
Total (Corr.)	20356.276	57			

Correlation Coefficient = 0.241335 R-squared = 5.82 percent
Stnd. Error of Est. = 18.5023

Three-D Histogram of SAT verbal aptitude scores vs intermediate algebra scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: IALG   Independent variable: SAT

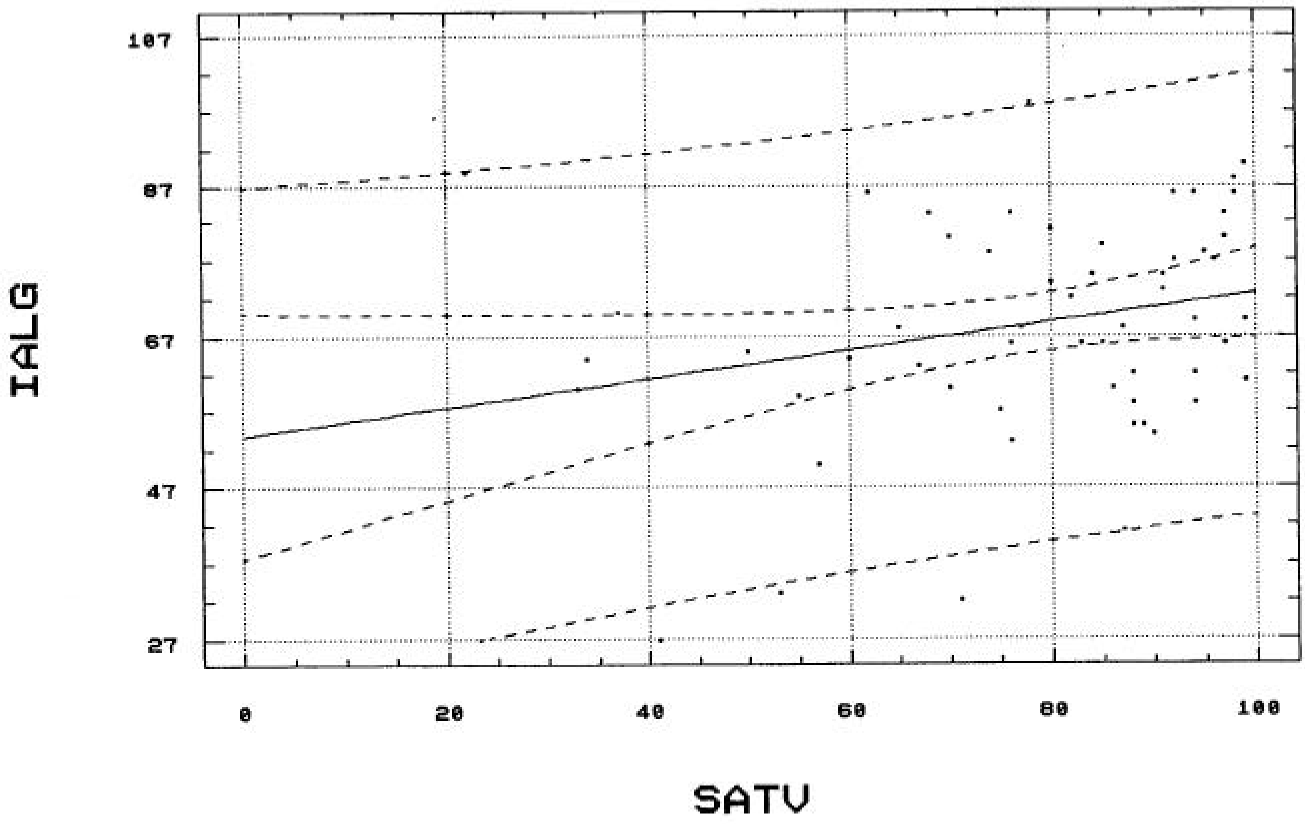
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	53.8058	8.08328	6.65643	.00000
Slope	0.187918	0.100977	1.861	.06800

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	718.84461	1	718.84461	3.4633	.06800
Error	11623.380	56	207.560		
Total (Corr.)	12342.224	57			

Correlation Coefficient = 0.241335   R-squared = 5.82 percent
Stnd. Error of Est. = 14.407

Regression of scores in intermediate algebra on SAT verbal aptitude scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: ANAL — Independent variable: SATV

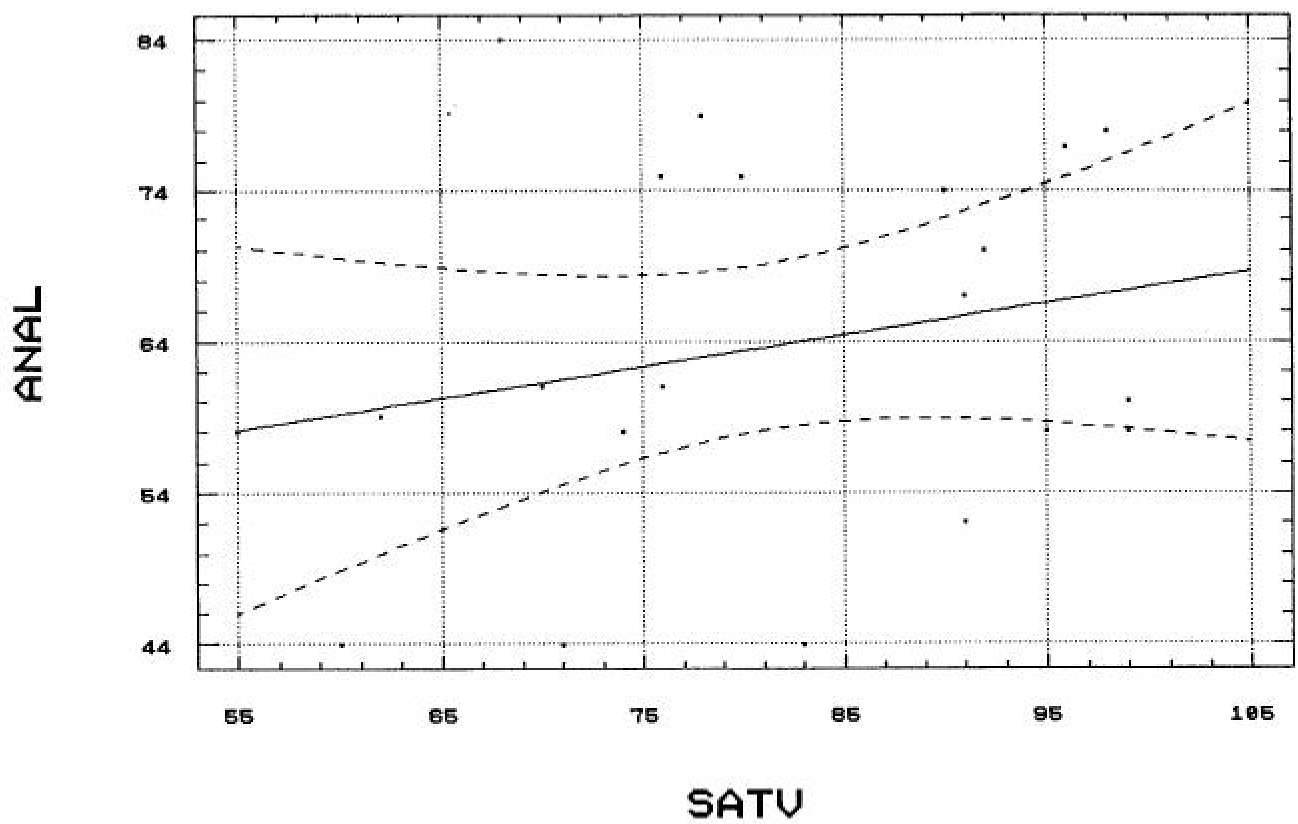
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	46.5655	16.1927	2.87571	.00968
Slope	0.210167	0.196912	1.06731	.29921

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	165.22090	1	165.22090	1.1392	.29921
Error	2755.7315	19	145.0385		
Total (Corr.)	2920.9524	20			

Correlation Coefficient = 0.237832 R-squared = 5.66 percent
Stnd. Error of Est. = 12.0432

Regression of analytic geometry scores on SAT verbal aptitude scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: SATV   Independent variable: ANAL

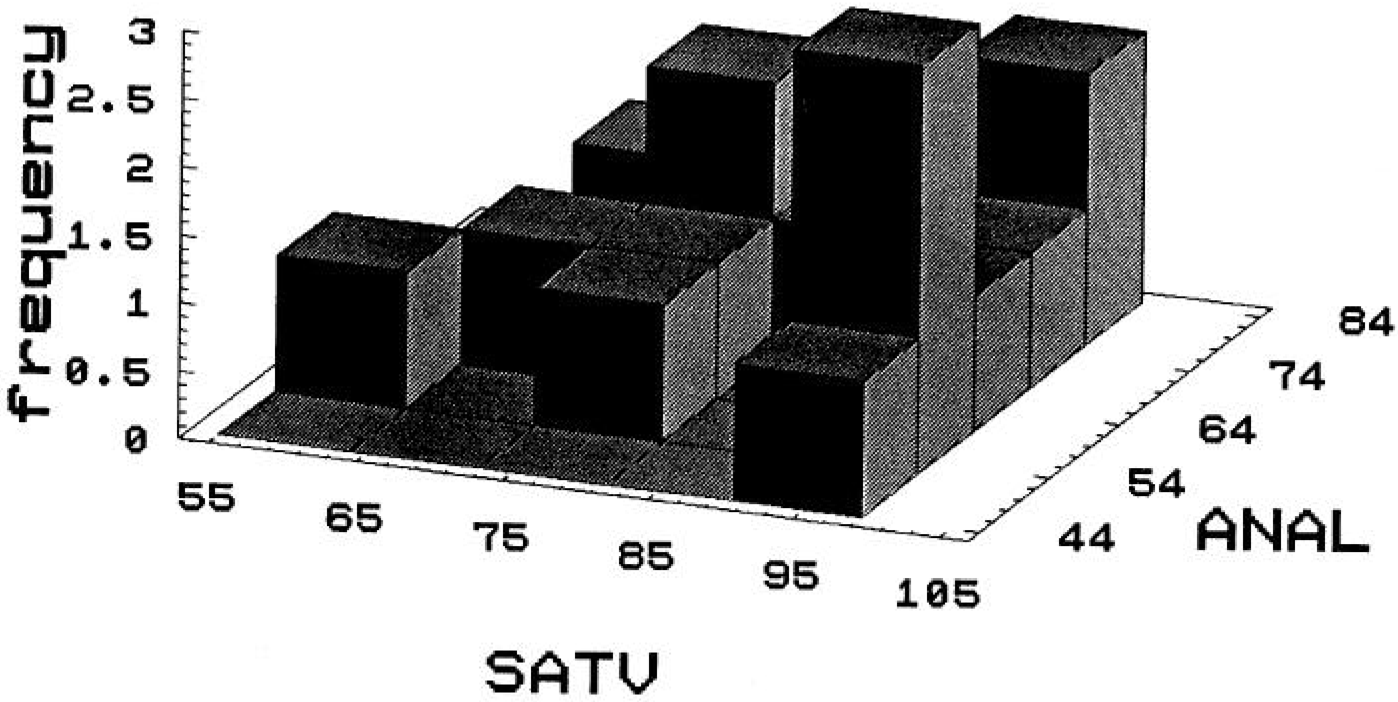
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	64.0205	16.3159	3.92381	.00091
Slope	0.269139	0.252166	1.06731	.29921

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	211.58188	1	211.58188	1.1392	.29921
Error	3528.9896	19	185.7363		
Total (Corr.)	3740.5714	20			

Correlation Coefficient = 0.237832   R-squared = 5.66 percent
Stnd. Error of Est. = 13.6285

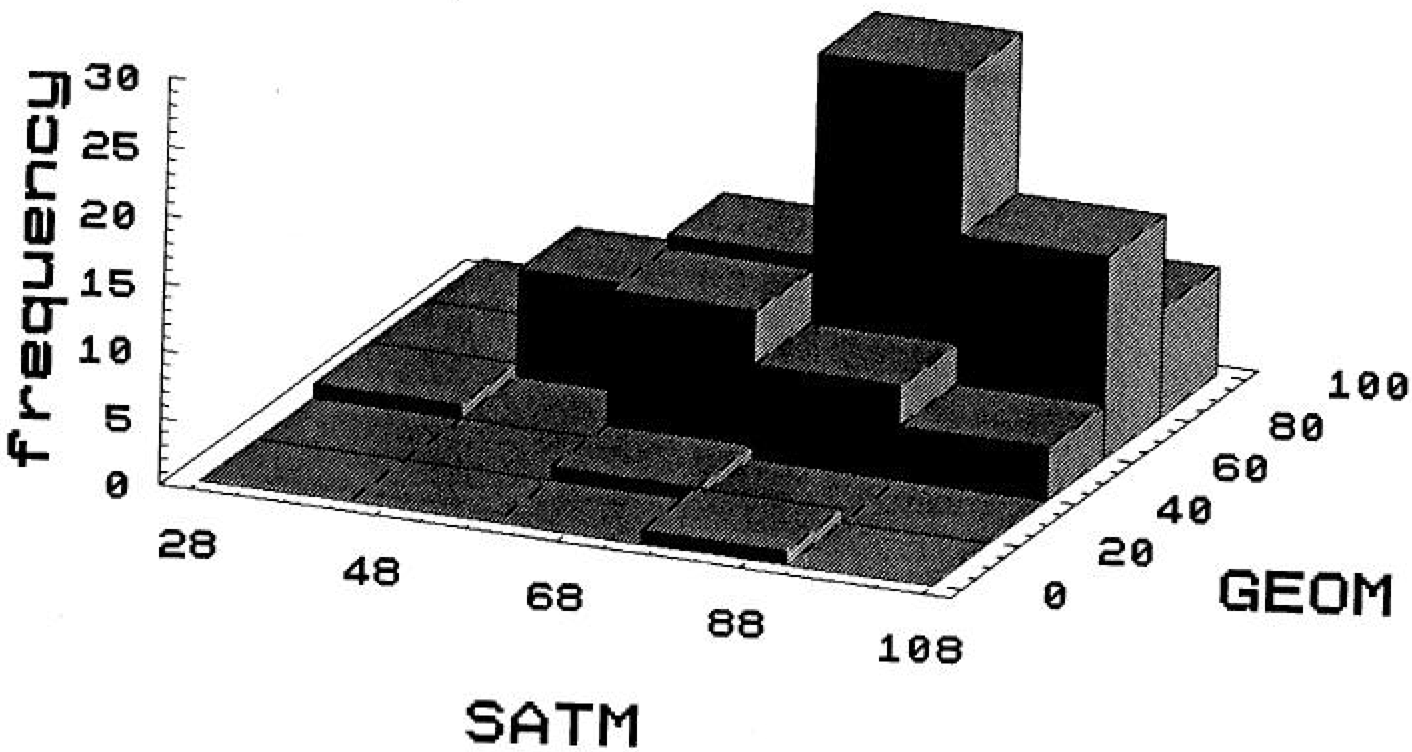
Three-D Histogram of SAT verbal aptitude scores vs analytic geometry scores



```
Regression Analysis - Linear model: Y = a+bX
-----------------------------------------------------------------------------
Dependent variable: SATM
                                                  Independent variable: GEOI
-----------------------------------------------------------------------------
Parameter        Estimate      Standard          T            Prob.
                                 Error         Value          Level
-----------------------------------------------------------------------------
Intercept         56.1211      8.42774        6.65909        .00000
Slope             0.362203     0.120885       2.99626        .00343
-----------------------------------------------------------------------------

                           Analysis of Variance
-----------------------------------------------------------------------------
Source               Sum of Squares    Df  Mean Square    F-Ratio  Prob. Level
Model                    2042.9233      1    2042.9233      8.978      .00343
Error                    23211.038    102      227.559
-----------------------------------------------------------------------------
Total (Corr.)            25253.962    103
-----------------------------------------------------------------------------
Correlation Coefficient = 0.284421
Stnd. Error of Est. = 15.0851                 R-squared =    8.09 percent
```

Three-D Histogram of SAT math aptitude scores vs elementary geometry scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: GEOM　　　Independent variable: SATM

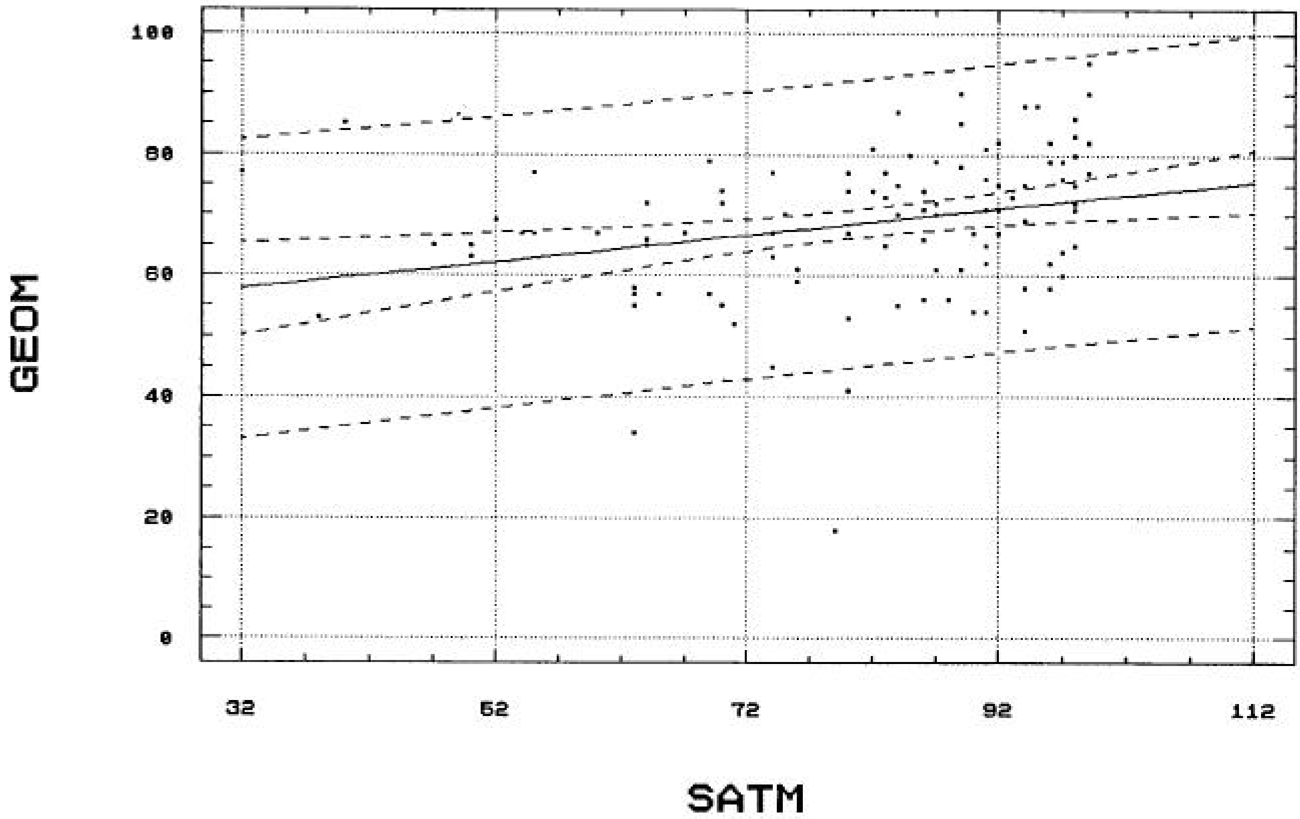
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	50.5482	6.14708	8.22313	.00000
Slope	0.223342	0.0745404	2.99626	.00343

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	1259.7088	1	1259.7088	8.978	.00343
Error	14312.407	102	140.318		
Total (Corr.)	15572.115	103			

Correlation Coefficient = 0.284421　　　R-squared = 8.09 percent
Stnd. Error of Est. = 11.8456

Regression of high school geometry scores on SAT scores in mathematics



Regression Analysis - Linear model: Y = a+bX

Dependent variable: EALG — Independent variable: SATM

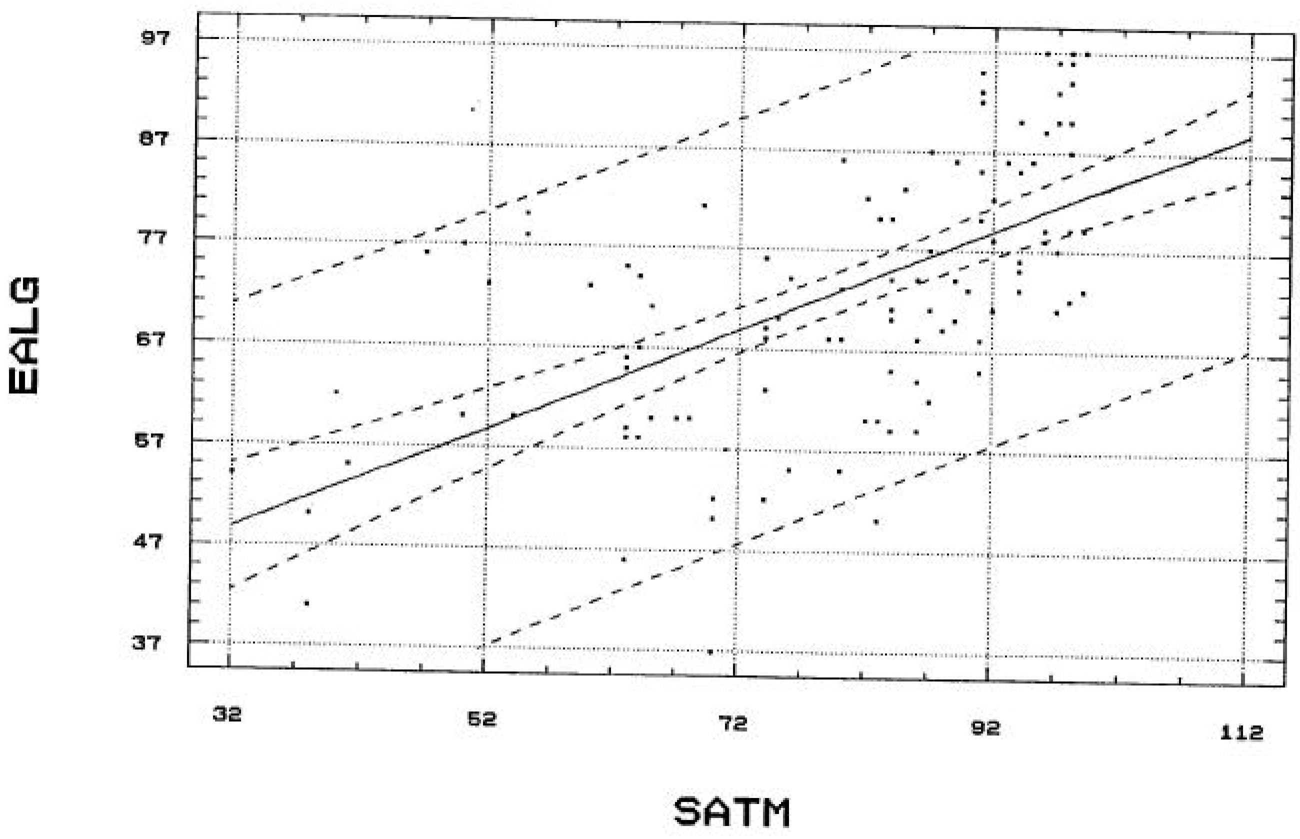
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	32.6453	5.08345	6.42189	.00000
Slope	0.502292	0.0622339	8.07104	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	7378.0996	1	7378.0996	65.142	.00000
Error	12345.594	109	113.262		
Total (Corr.)	19723.694	110			

Correlation Coefficient = 0.611615 — R-squared = 37.41 percent
Stnd. Error of Est. = 10.6425

Regression of high school algebra scores on SAT scores in mathematics



Regression Analysis - Linear model: Y = a+bX

Dependent variable: SATM   Independent variable: EALG

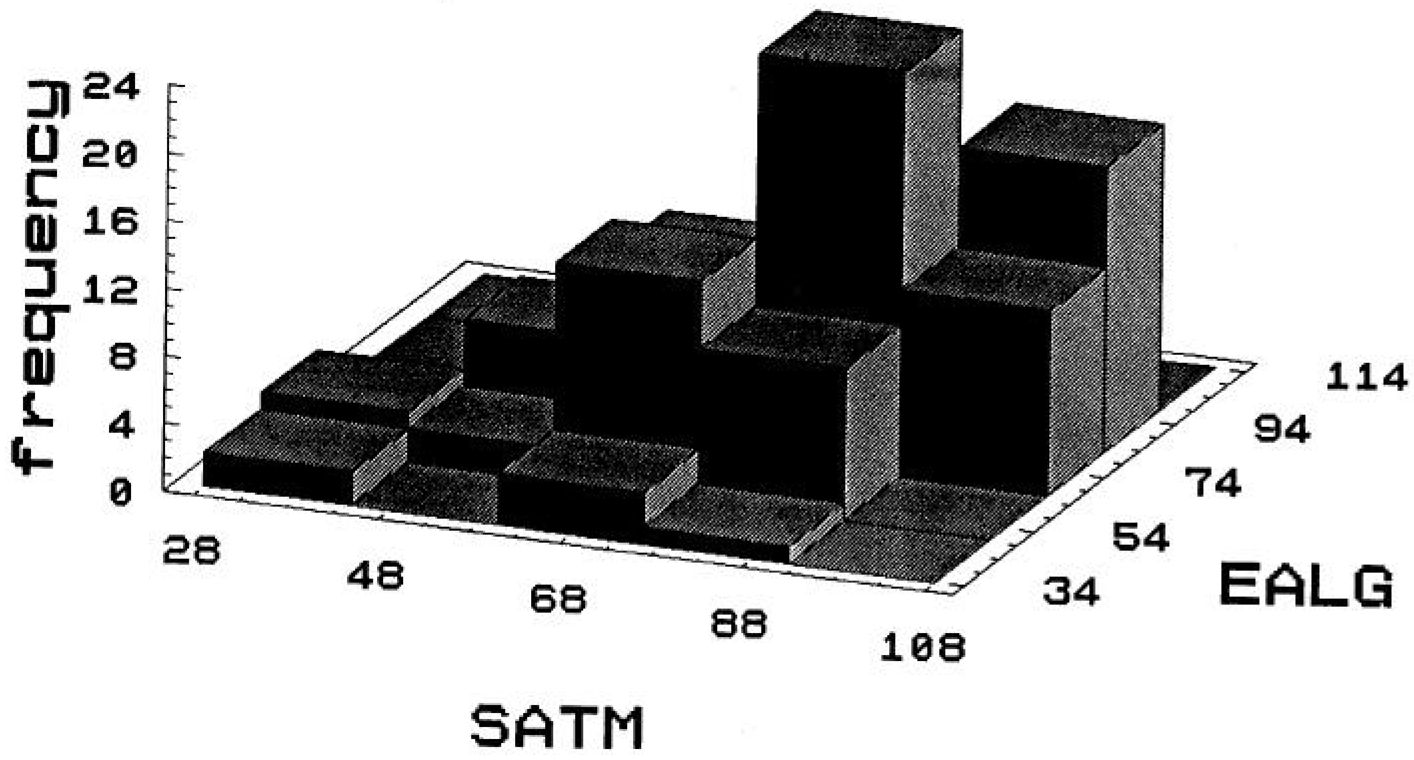
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	25.796	6.83416	3.77456	.00026
Slope	0.744732	0.0922721	8.07104	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	10939.267	1	10939.267	65.14	.00000
Error	18304.409	109	167.930		
Total (Corr.)	29243.676	110			

Correlation Coefficient = 0.611615   R-squared = 37.41 percent
Stnd. Error of Est. = 12.9588

Three-D Histogram of SAT math aptitude scores vs elementary algebra scores



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egression Analysis - Linear model: Y = a+bX
-----------------------------------------------------------------------------
ependent variable: SATM                      Independent variable: TRIG
-----------------------------------------------------------------------------
                          Standard           T              Prob.
arameter      Estimate    Error              Value          Level
-----------------------------------------------------------------------------
ntercept       59.0566     6.65375           8.87569        .00000
lope           0.368533    0.0975804         3.77671        .00030
-----------------------------------------------------------------------------

                      Analysis of Variance
-----------------------------------------------------------------------------
ource             Sum of Squares    Df   Mean Square   F-Ratio  Prob. Level
odel                   2552.9063     1     2552.9063    14.264       .00030
rror                   14497.479    81       178.981
-----------------------------------------------------------------------------
otal (Corr.)           17050.386    82

orrelation Coefficient = 0.386946          R-squared =   14.97 percent
tnd. Error of Est. = 13.3784
```
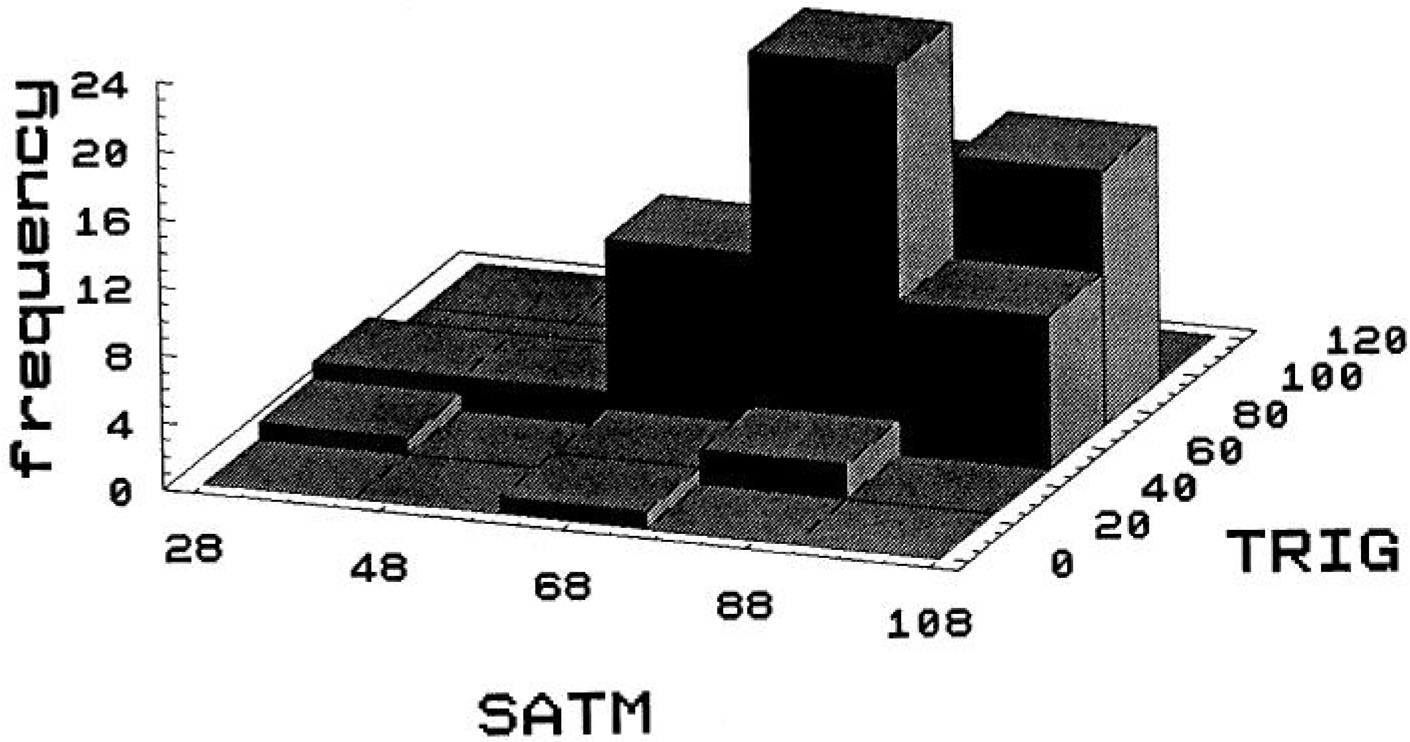

Three-D Histogram of SAT math aptitude scores vs elementary trigonometry scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: TRIG   Independent variable: SAT

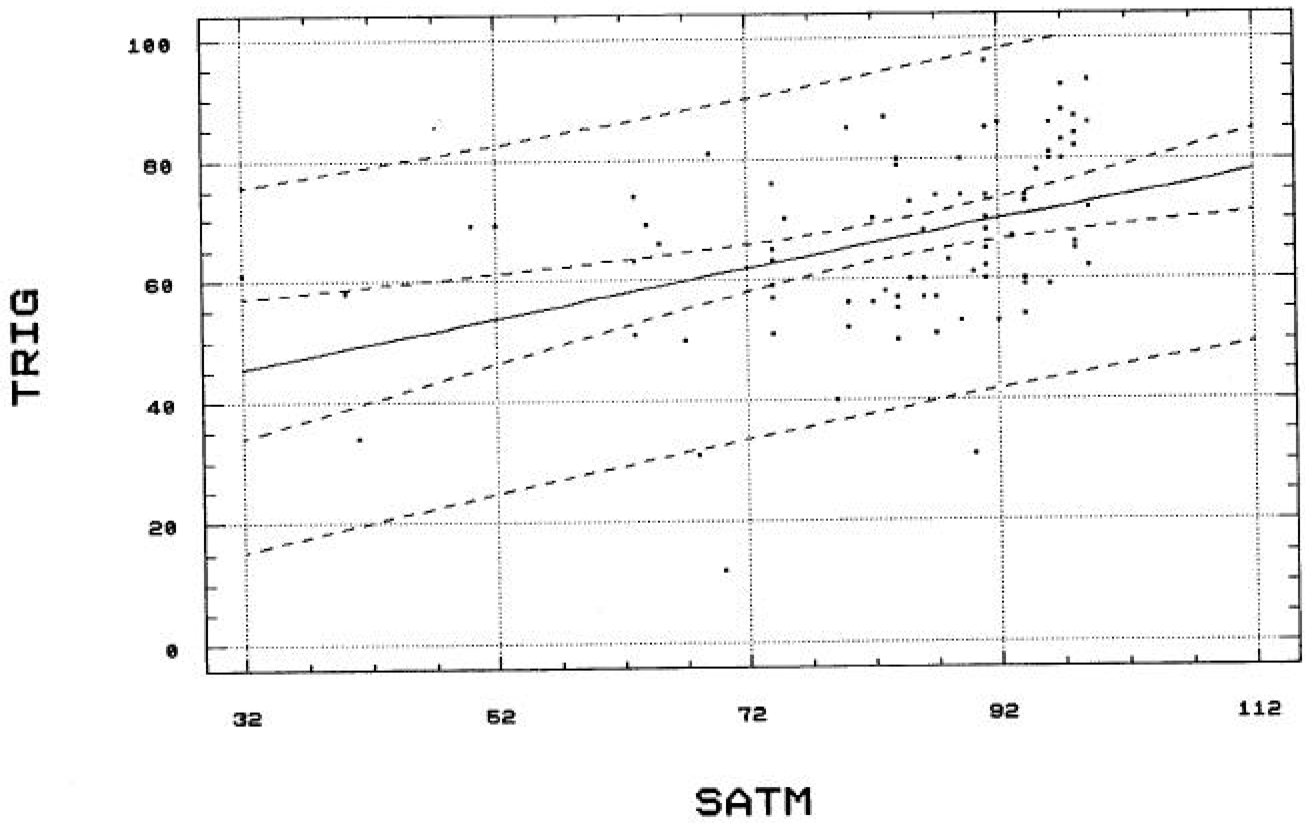
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	32.5548	9.1209	3.56925	.00061
Slope	0.406279	0.107575	3.77671	.00030

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	2814.3841	1	2814.3841	14.264	.00030
Error	15982.363	81	197.313		
Total (Corr.)	18796.747	82			

Correlation Coefficient = 0.386946   R-squared = 14.97 percent
Stnd. Error of Est. = 14.0468

Regression of high school trigonometry scores on SAT scores in mathematics



Regression Analysis - Linear model: Y = a+bX

Dependent variable: IALG   Independent variable: SA

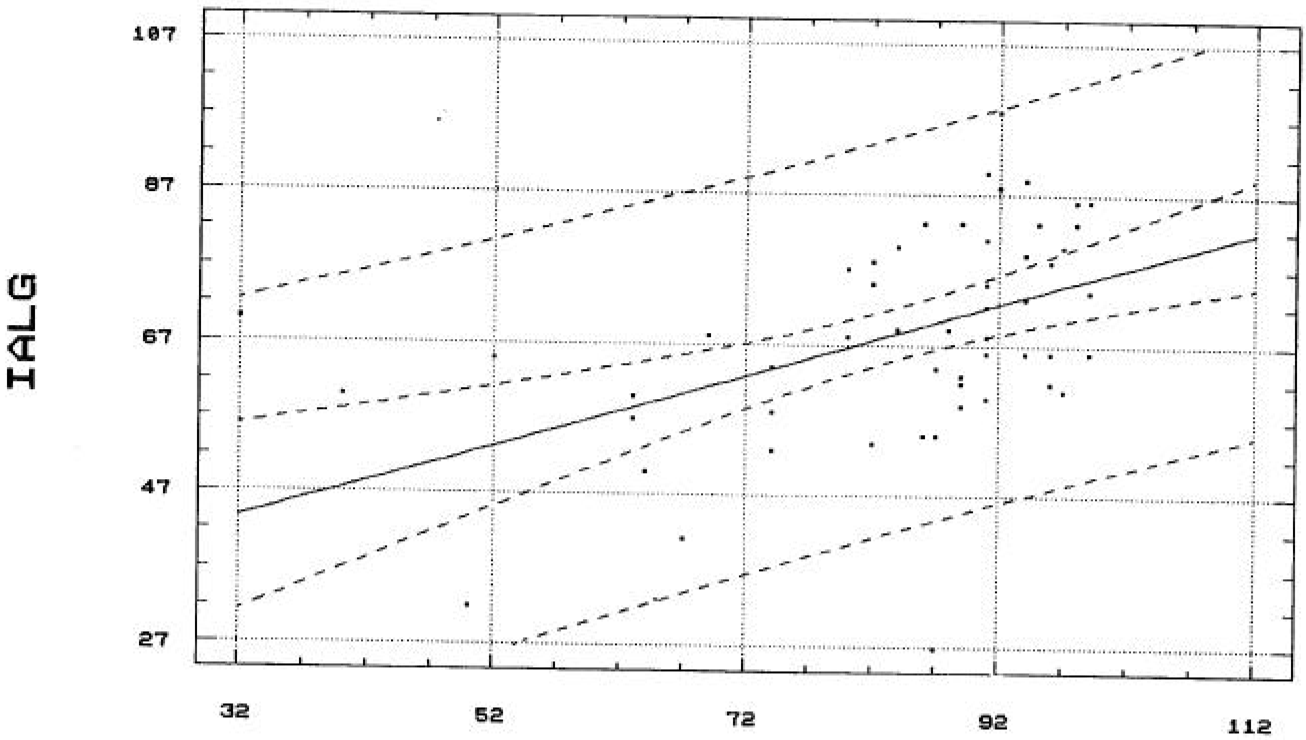
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	28.3472	9.73726	2.91121	.00516
Slope	0.47886	0.114535	4.18089	.00010

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	2936.0456	1	2936.0456	17.480	.00010
Error	9406.1786	56	167.9675		
Total (Corr.)	12342.224	57			

Correlation Coefficient = 0.487736   R-squared = 23.79 percent
Stnd. Error of Est. = 12.9602

Regression of intermediate algebra scores on SAT scores in mathematics



Regression Analysis - Linear model: Y = a+bX

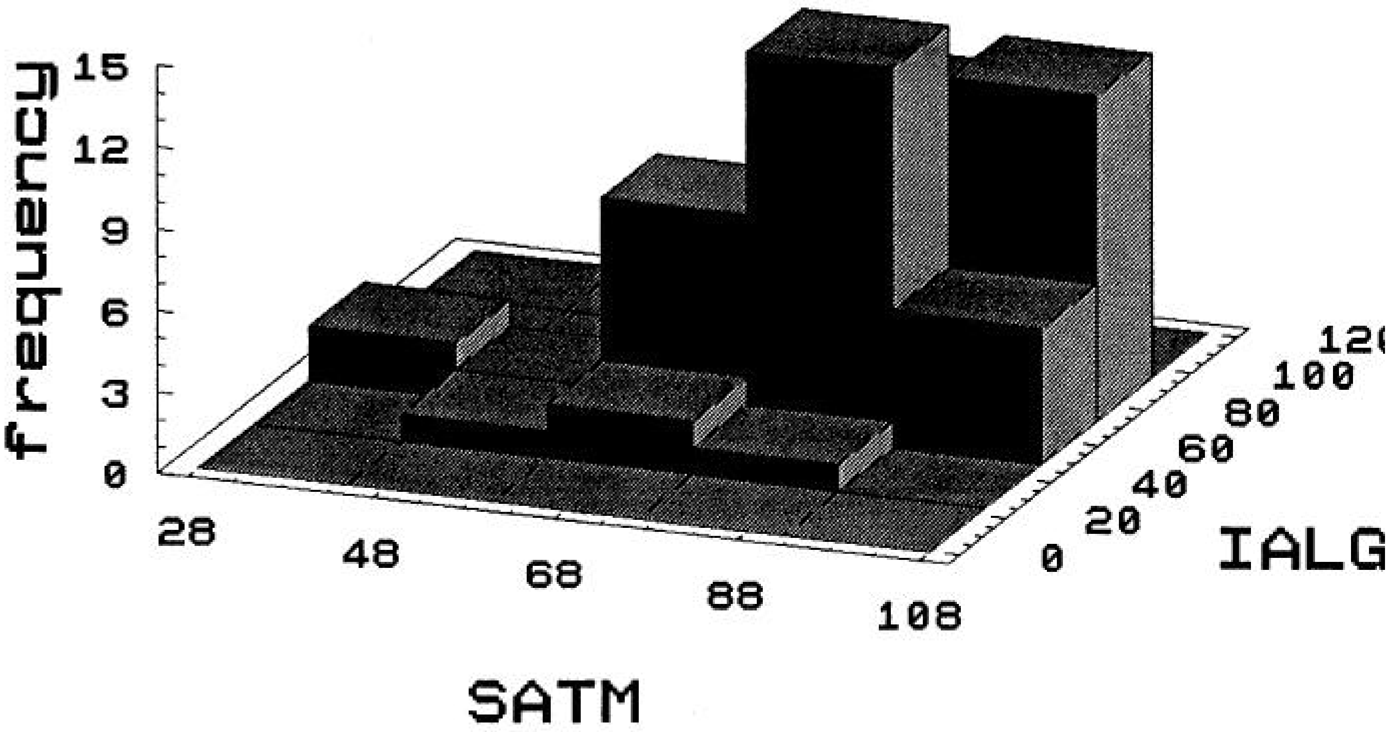
Dependent variable: SATM				Independent variable: IAL
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	49.712	8.31372	5.97951	.00000
Slope	0.496777	0.118821	4.18089	.00010

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	3045.8998	1	3045.8998	17.480	.00010
Error	9758.1174	56	174.2521		
Total (Corr.)	12804.017	57			

Correlation Coefficient = 0.487736 R-squared = 23.79 percent
Stnd. Error of Est. = 13.2005

Three-D Histogram of SAT math aptitude scores vs intermediate algebra scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: SATM   Independent variable: ANAL

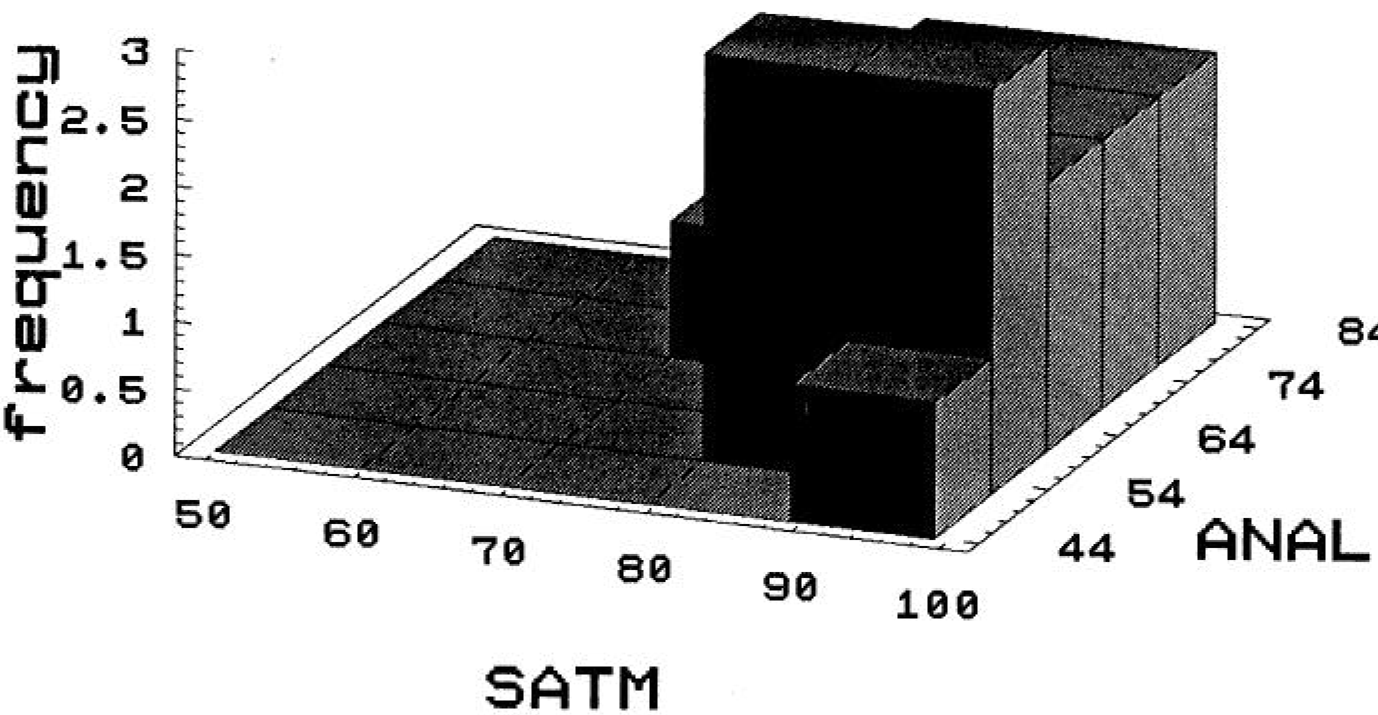
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	78.2376	13.4191	5.83034	.00001
Slope	0.150456	0.207395	0.725459	.47701

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	66.122037	1	66.122037	.52629	.47701
Error	2387.1161	19	125.6377		
Total (Corr.)	2453.2381	20			

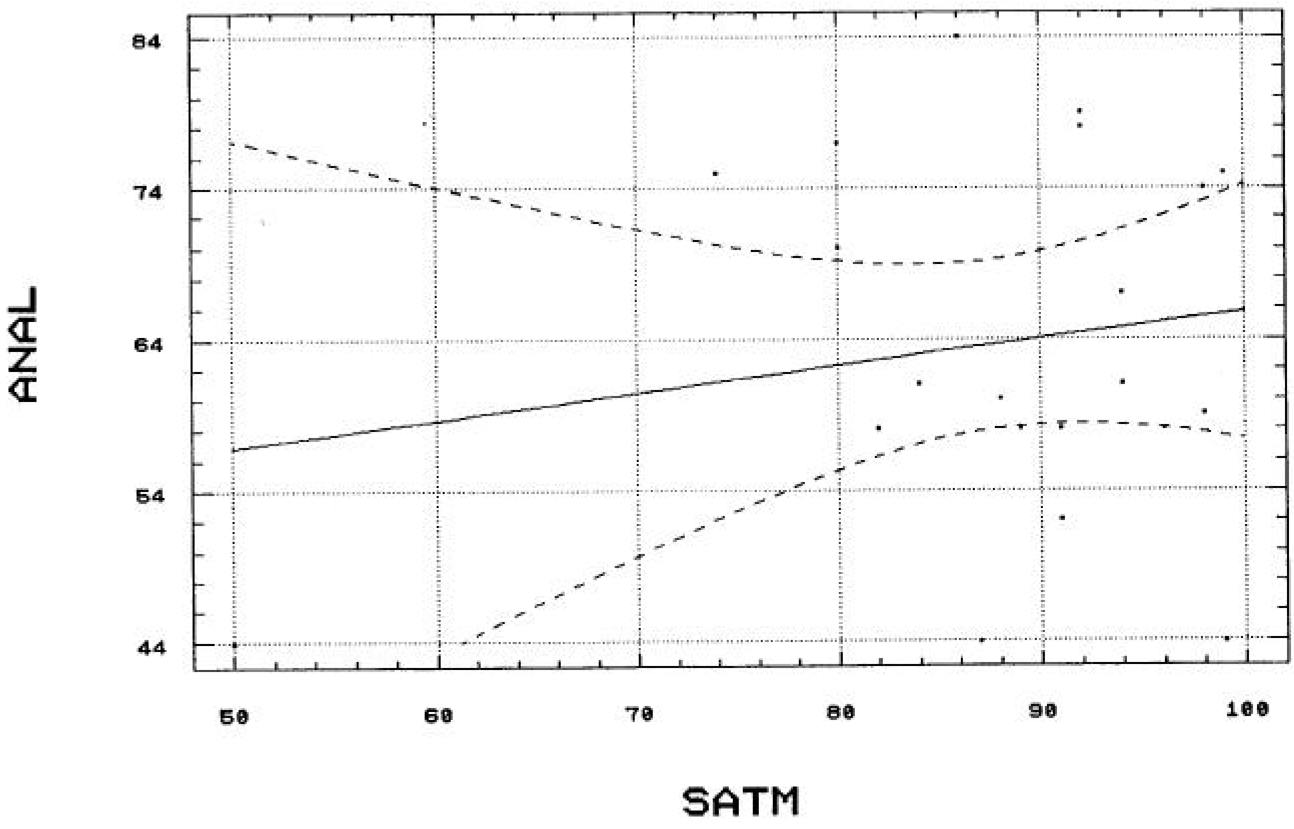
Correlation Coefficient = 0.164174   R-squared =   2.70 percent
Stnd. Error of Est. = 11.2088

Three-D Histogram of SAT math aptitude scores vs analytic geometry scores



```
Regression Analysis - Linear model: Y = a+bX
-----------------------------------------------------------------------------
Dependent variable: ANAL                        Independent variable: SAT
-----------------------------------------------------------------------------
                                Standard            T             Prob.
Parameter         Estimate        Error            Value           Level
-----------------------------------------------------------------------------
Intercept          47.8887       21.8469          2.19202         .04104
Slope             0.179141      0.246935         0.725459         .47701
-----------------------------------------------------------------------------

                             Analysis of Variance
-----------------------------------------------------------------------------
Source               Sum of Squares     Df  Mean Square    F-Ratio  Prob. Level
Model                     78.728323      1    78.728323     .52629       .47701
Error                     2842.2241     19     149.5907
-----------------------------------------------------------------------------
Total (Corr.)             2920.9524     20

Correlation Coefficient = 0.164174          R-squared =   2.70 percent
Stnd. Error of Est. = 12.2307
```

Regression of ANALytic geometry scores on SAT scores in mathematics



Regression Analysis - Linear model: Y = a+bX

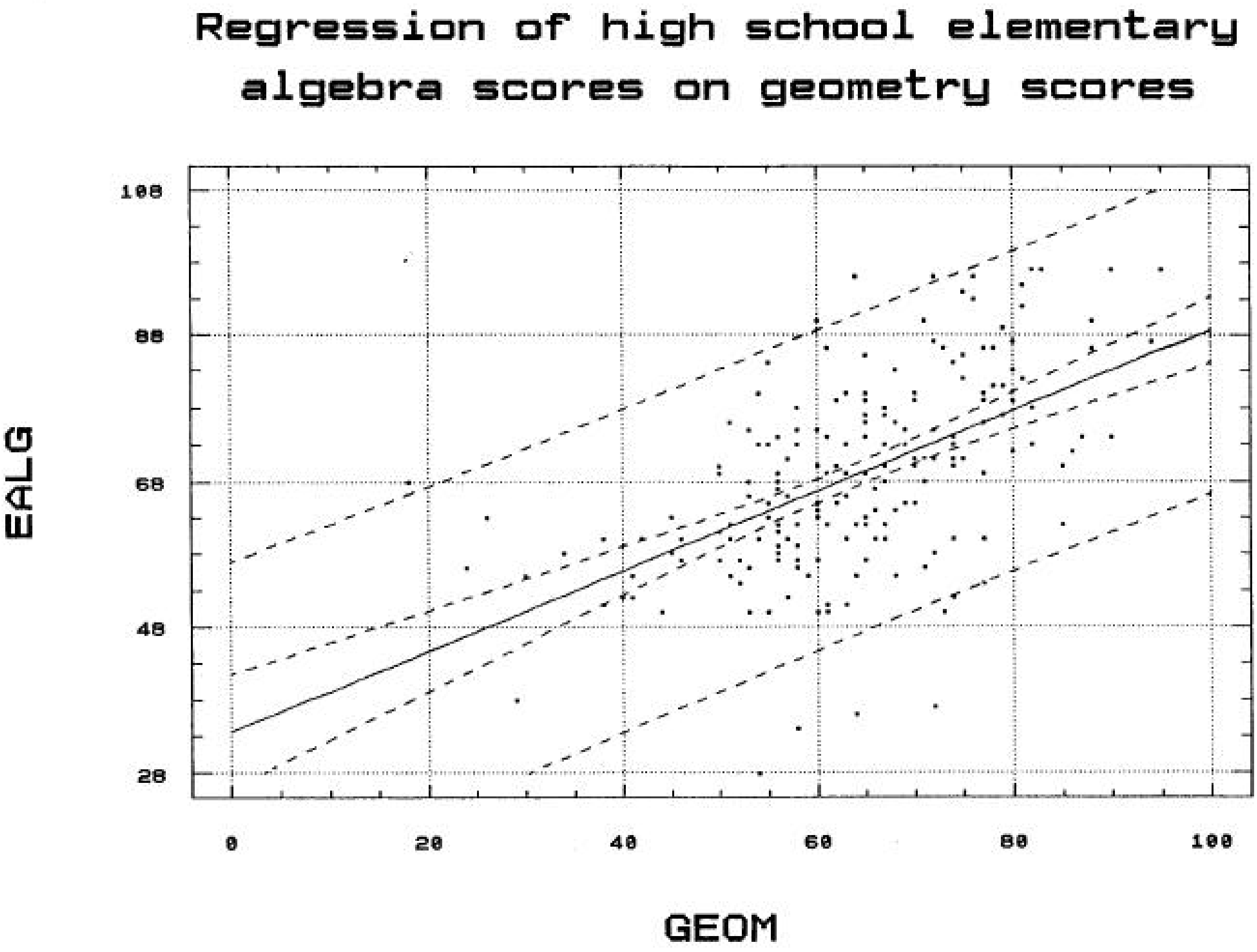
Dependent variable: EALG Independent variable: GI

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	33.6897	3.98305	8.45828	.00000
Slope	0.549302	0.0608994	9.01983	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	10005.234	1	10005.234	81.36	.00000
Error	23242.997	189	122.979		
Total (Corr.)	33248.230	190			

Correlation Coefficient = 0.548567 R-squared = 30.09 percent
Stnd. Error of Est. = 11.0896



Regression Analysis - Linear model: Y = a+bX

Dependent variable: GEOM — Independent variable: EAL

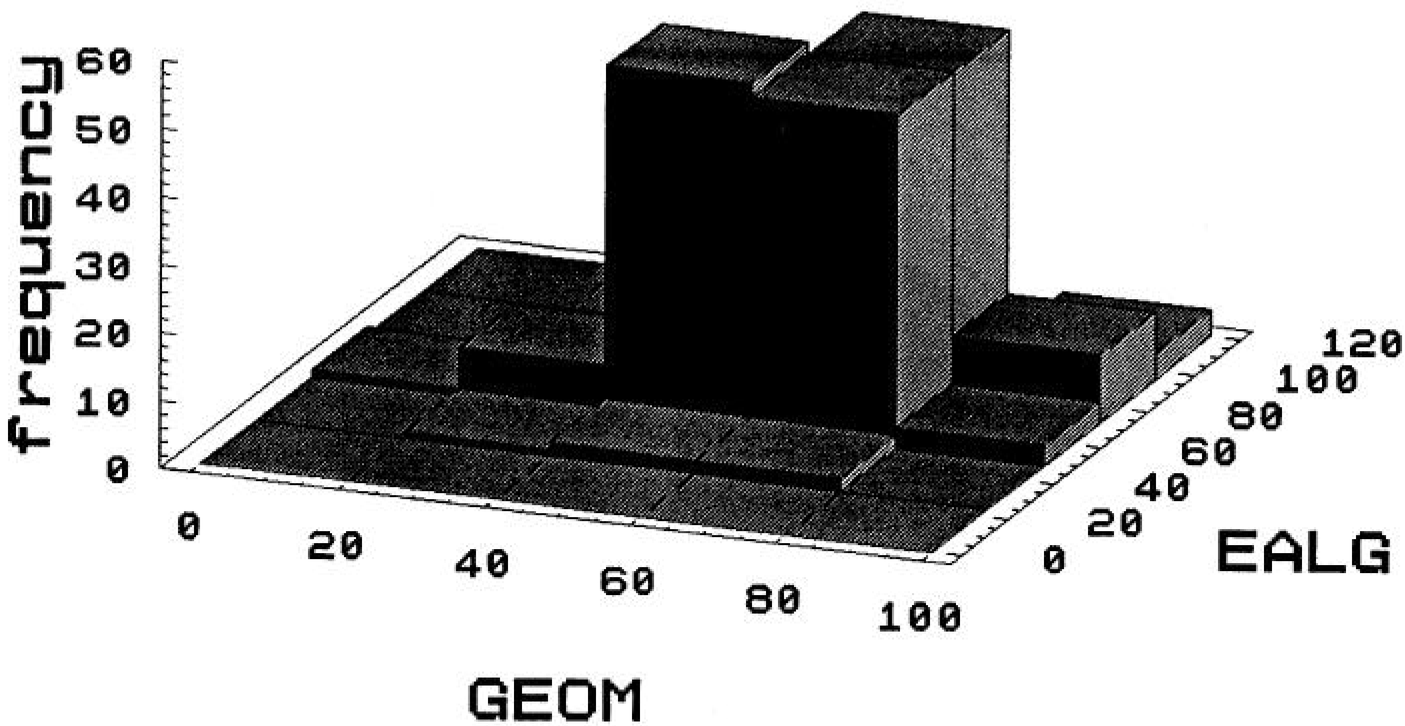
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	26.3284	4.25955	6.18102	.00000
Slope	0.547832	0.0607364	9.01983	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	9978.4561	1	9978.4561	81.357	.00000
Error	23180.790	189	122.650		
Total (Corr.)	33159.246	190			

Correlation Coefficient = 0.548567 R-squared = 30.09 percent
Stnd. Error of Est. = 11.0747

Three-D Histogram of elementary GEOMetry scores vs elementary algebra scores



egression Analysis - Linear model: Y = a+bX

ependent variable: GEOM Independent variable: TRIG

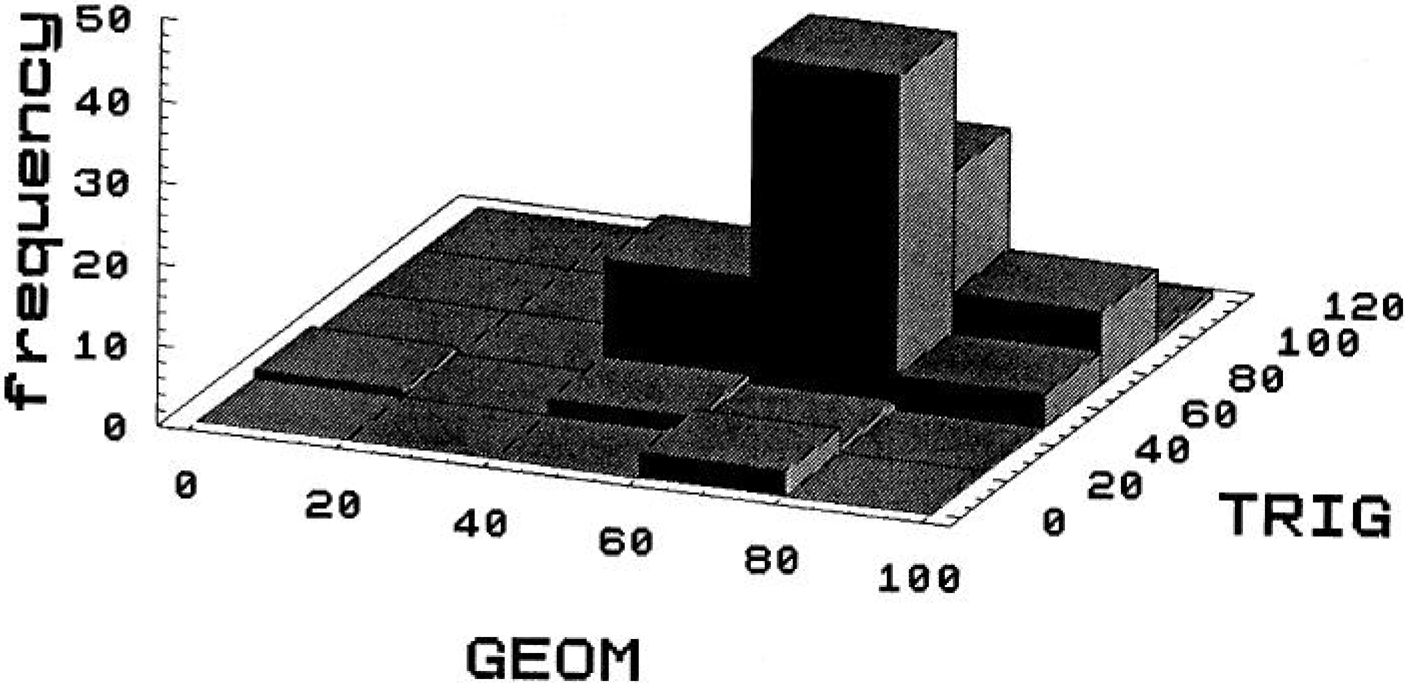
arameter	Estimate	Standard Error	T Value	Prob. Level
ntercept	49.917	4.22642	11.8107	.00000
lope	0.30134	0.0620304	4.85795	.00000

Analysis of Variance

ource	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
odel	2415.8463	1	2415.8463	23.600	.00000
rror	10441.529	102	102.368		
otal (Corr.)	12857.375	103			

orrelation Coefficient = 0.433469 R-squared = 18.79 percent
tnd. Error of Est. = 10.1177

Three-D Histogram of elementary GEOMetry scores vs elementary trigonometry scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: TRIG   Independent variable: GEOM

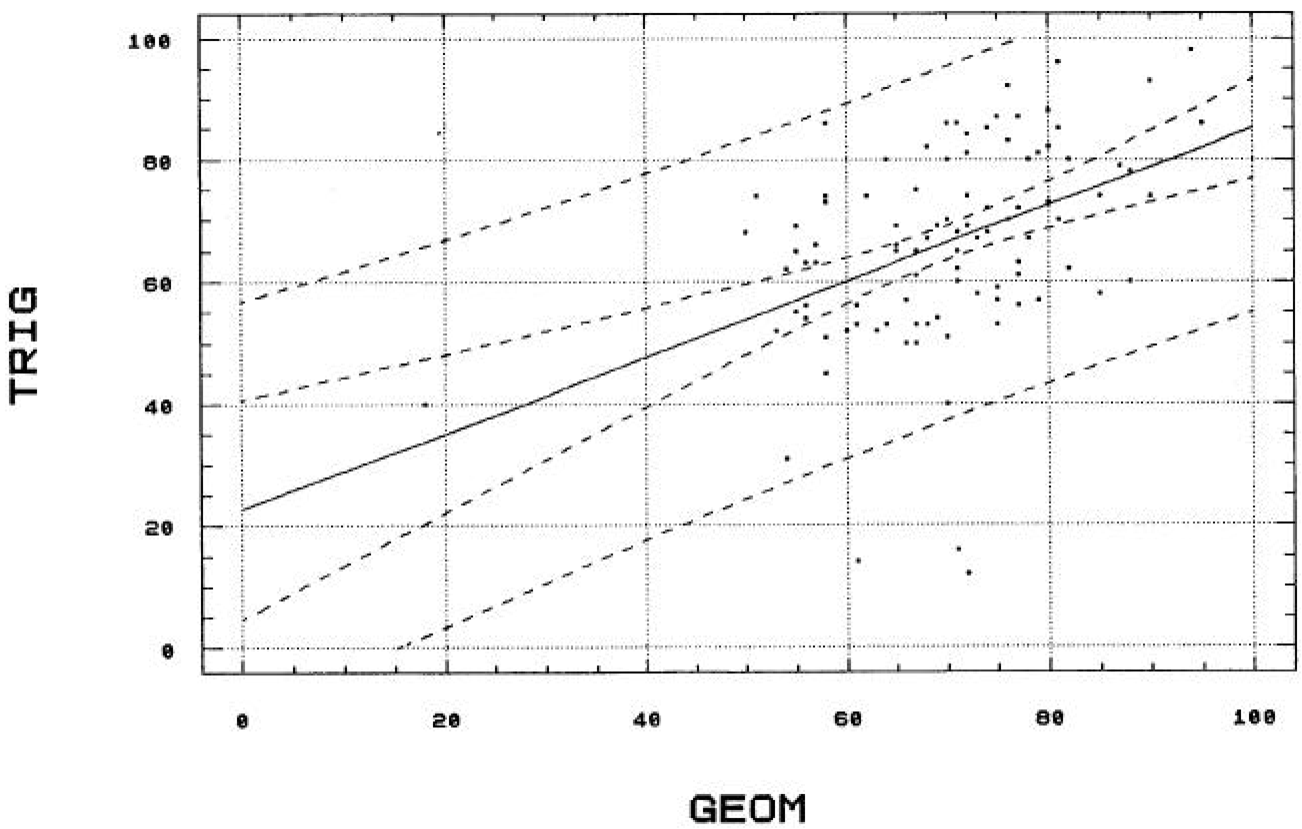
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	22.6614	9.08152	2.49533	.01419
Slope	0.623533	0.128353	4.85795	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	4998.8656	1	4998.8656	23.600	.00000
Error	21605.596	102	211.820		
Total (Corr.)	26604.462	103			

Correlation Coefficient = 0.433469   R-squared = 18.79 percent
Stnd. Error of Est. = 14.554

Regression of high school trigonometry scores on elementary geometry scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: IALG | Independent variable: GEOM

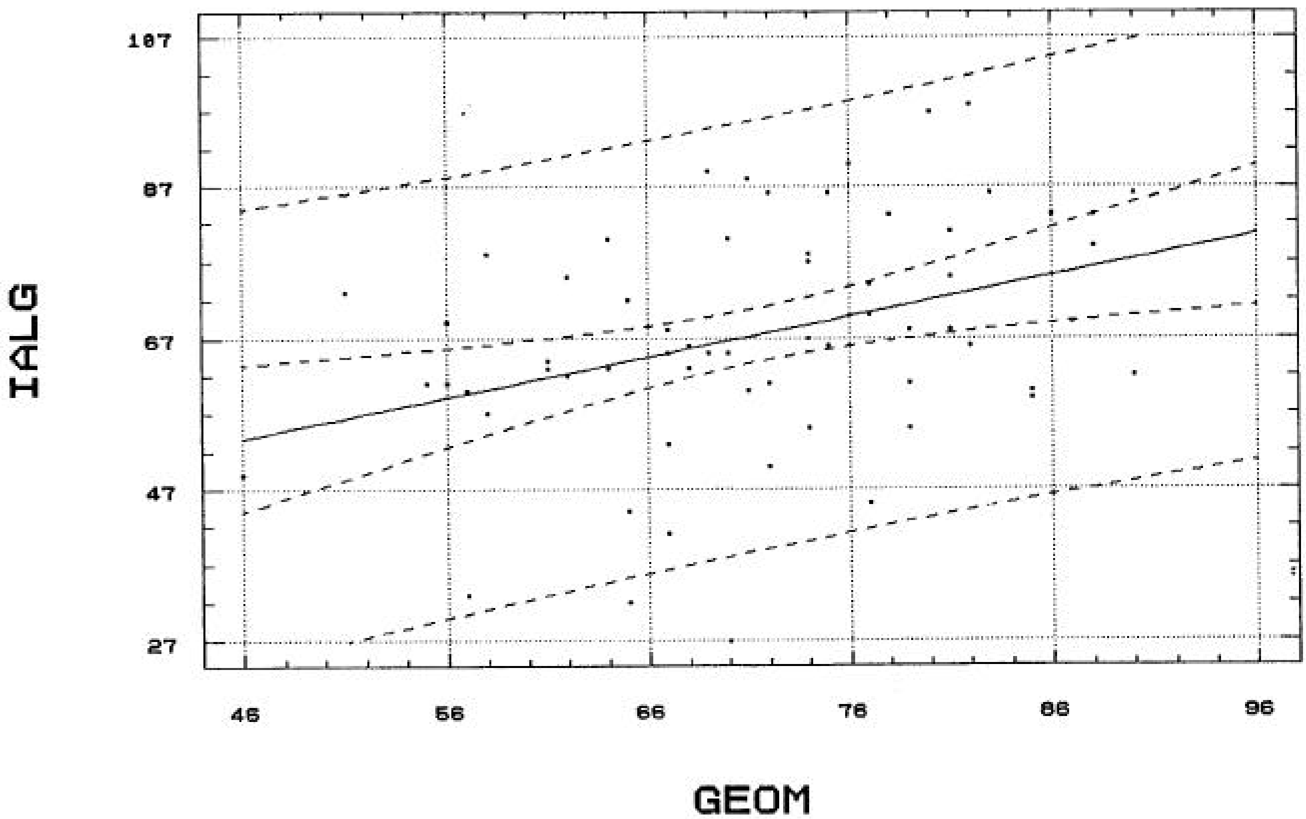
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	29.1352	12.7371	2.28743	.02565
Slope	0.535362	0.176163	3.03901	.00349

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	1858.7605	1	1858.7605	9.236	.00349
Error	12276.890	61	201.260		
Total (Corr.)	14135.651	62			

Correlation Coefficient = 0.362622 R-squared = 13.15 percent
Stnd. Error of Est. = 14.1866

Regression of intermediate algebra scores on scores in elementary geometry



Regression Analysis - Linear model: Y = a+bX

Dependent variable: GEOM   Independent variable: IAL

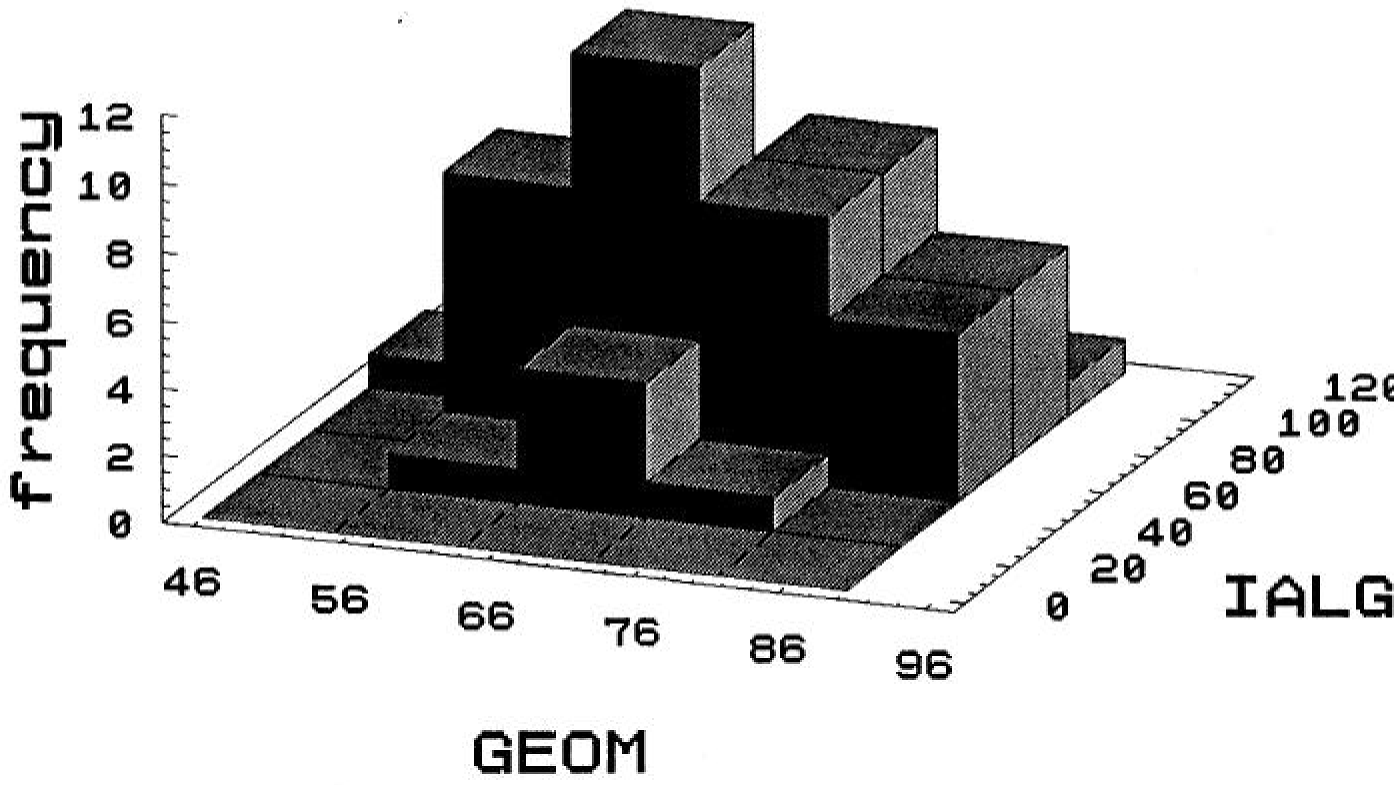
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	55.0178	5.58504	9.85093	.00000
Slope	0.245618	0.0808216	3.03901	.00349

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	852.77740	1	852.77740	9.2356	.00349
Error	5632.4924	61	92.3359		
Total (Corr.)	6485.2698	62			

Correlation Coefficient = 0.362622   R-squared = 13.15 percent
Stnd. Error of Est. = 9.60916

Three-D Histogram of elementary GEOMetry scores vs intermediate algebra scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: GEOM

Independent variable: AN

Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	57.0988	9.30754	6.13468	.00000
Slope	0.214541	0.150008	1.4302	.16611

Analysis of Variance

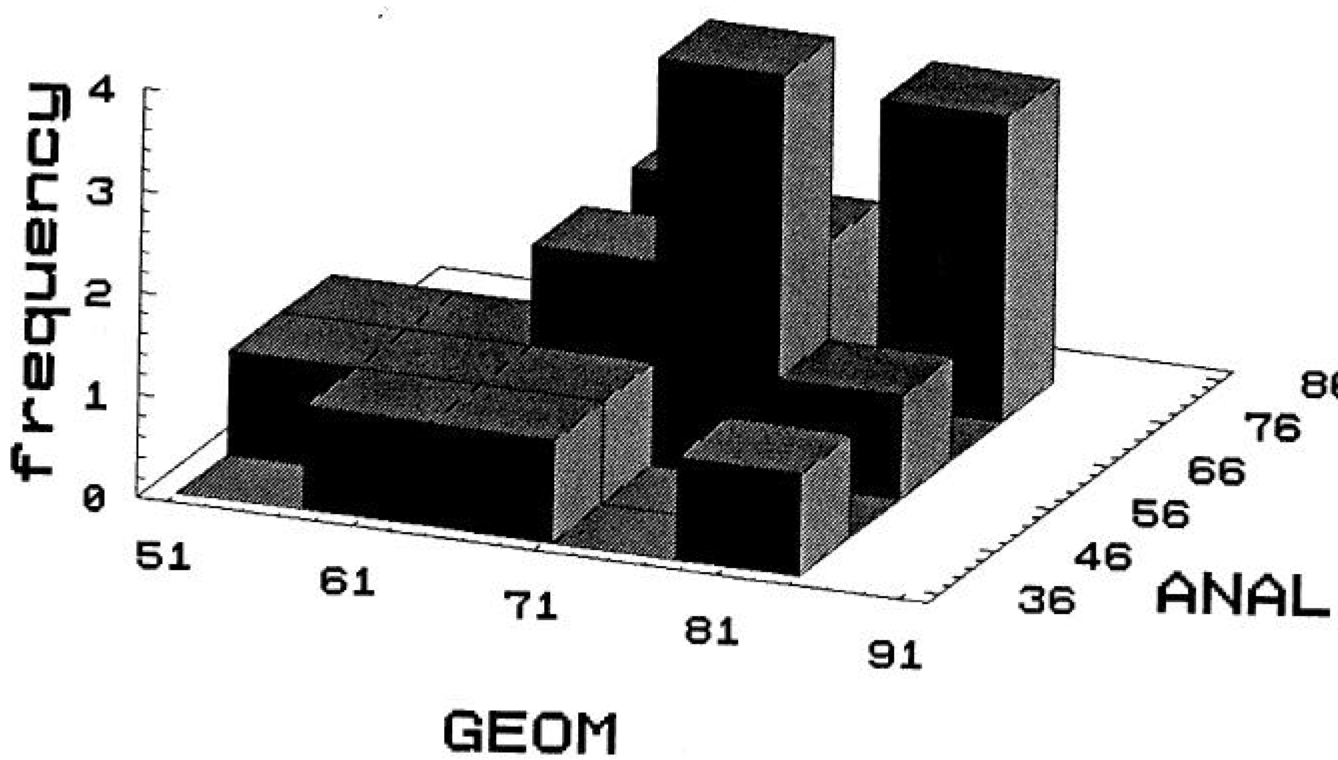
Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	165.08472	1	165.08472	2.0455	.16611
Error	1856.2753	23	80.7076		
Total (Corr.)	2021.3600	24			

Correlation Coefficient = 0.28578

Stnd. Error of Est. = 8.98374

R-squared = 8.17 percent

Three-D Histogram of elementary GEOMetry scores vs analytic geometry scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: ANAL Independent variable: GEC

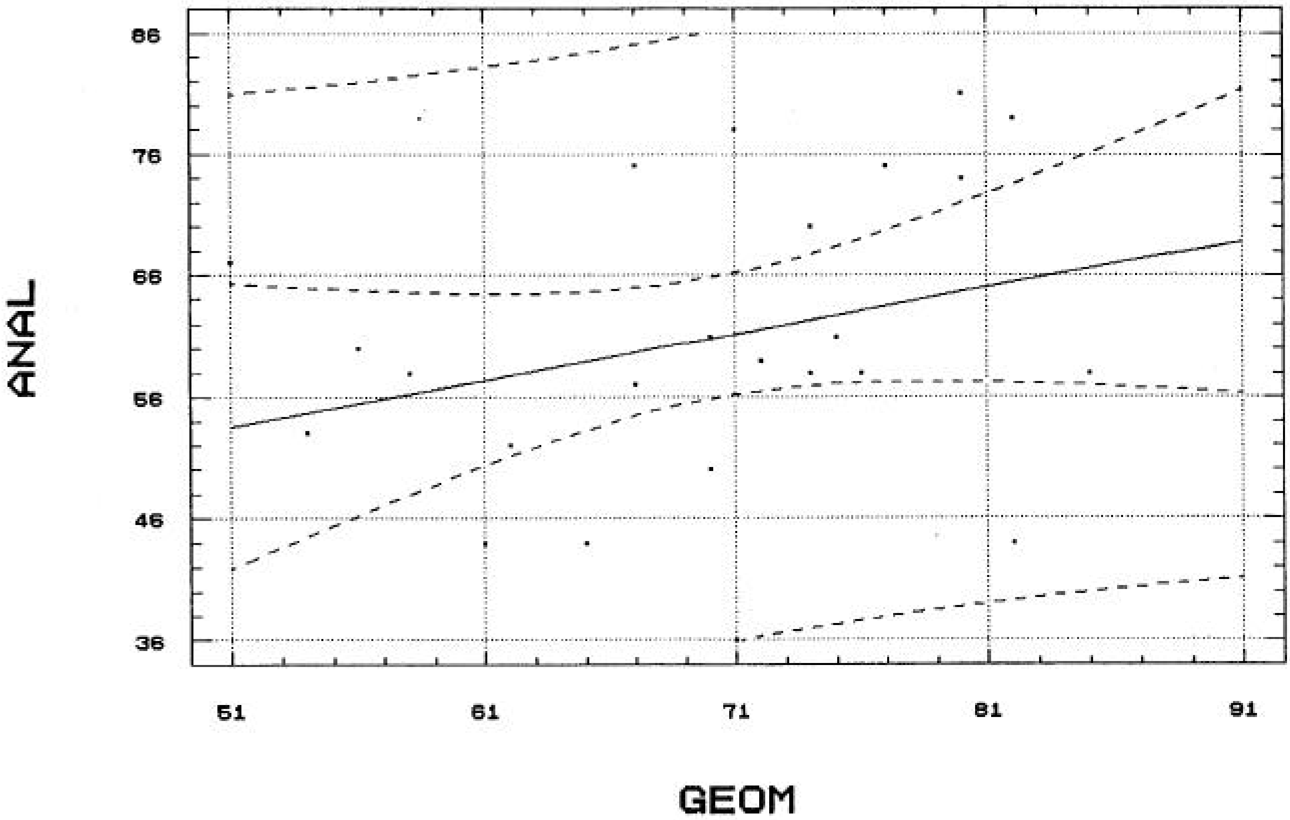
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	34.1719	18.8272	1.81503	.08258
Slope	0.380674	0.266169	1.4302	.16611

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	292.92134	1	292.92134	2.0455	.16611
Error	3293.7187	23	143.2052		
Total (Corr.)	3586.6400	24			

Correlation Coefficient = 0.28578 R-squared = 8.17 percent
Stnd. Error of Est. = 11.9668

Regression of ANALytic geometry scores on scores in elementary geometry



Regression Analysis - Linear model: Y = a+bX

Dependent variable: TRIG Independent variable: EAL

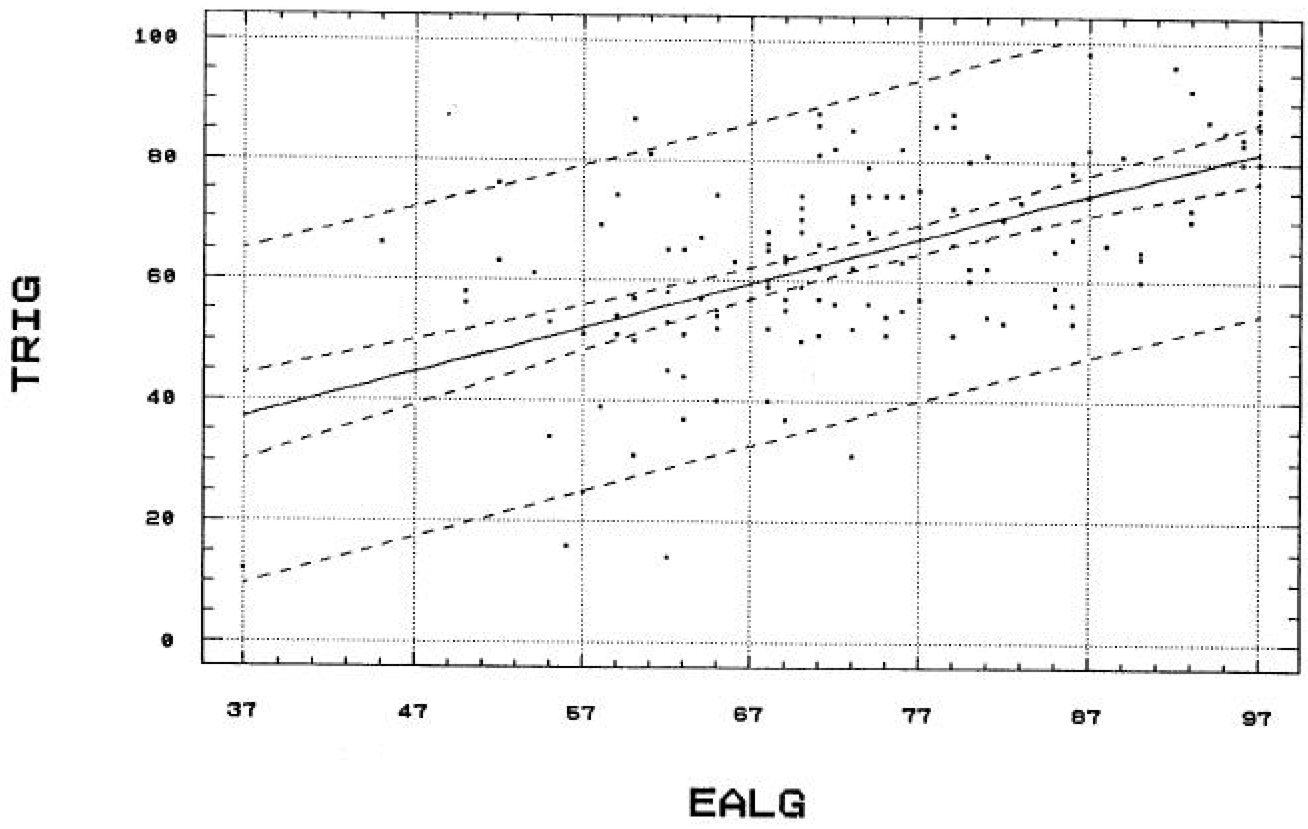
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	9.81088	6.97411	1.40676	.16181
Slope	0.740608	0.0941725	7.86438	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	11271.621	1	11271.621	61.85	.00000
Error	24420.937	134	182.246		
Total (Corr.)	35692.559	135			

Correlation Coefficient = 0.561959 R-squared = 31.58 percent
Stnd. Error of Est. = 13.4998

Regression of high school trigonometry scores on scores in elementary algebra



Regression Analysis - Linear model: Y = a+bX

Dependent variable: EALG Independent variable: TRIG

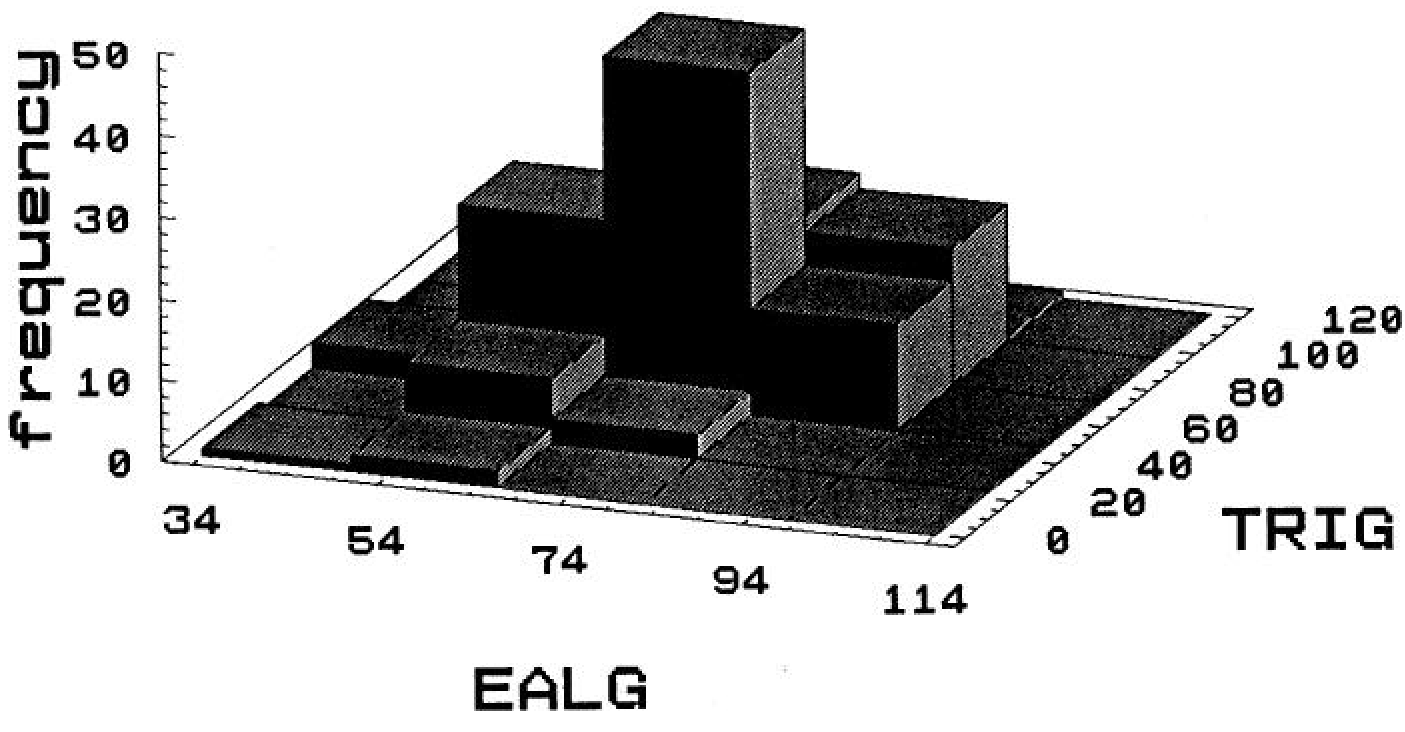
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	45.7835	3.57408	12.8099	.00000
Slope	0.426403	0.0542195	7.86438	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	6489.6018	1	6489.6018	61.848	.00000
Error	14060.281	134	104.927		
Total (Corr.)	20549.882	135			

Correlation Coefficient = 0.561959 R-squared = 31.58 percent
Stnd. Error of Est. = 10.2434

Three-D Histogram of elementary algebra scores vs elementary trigonometry scores



egression Analysis - Linear model: Y = a+bX

ependent variable: EALG Independent variable: IALG

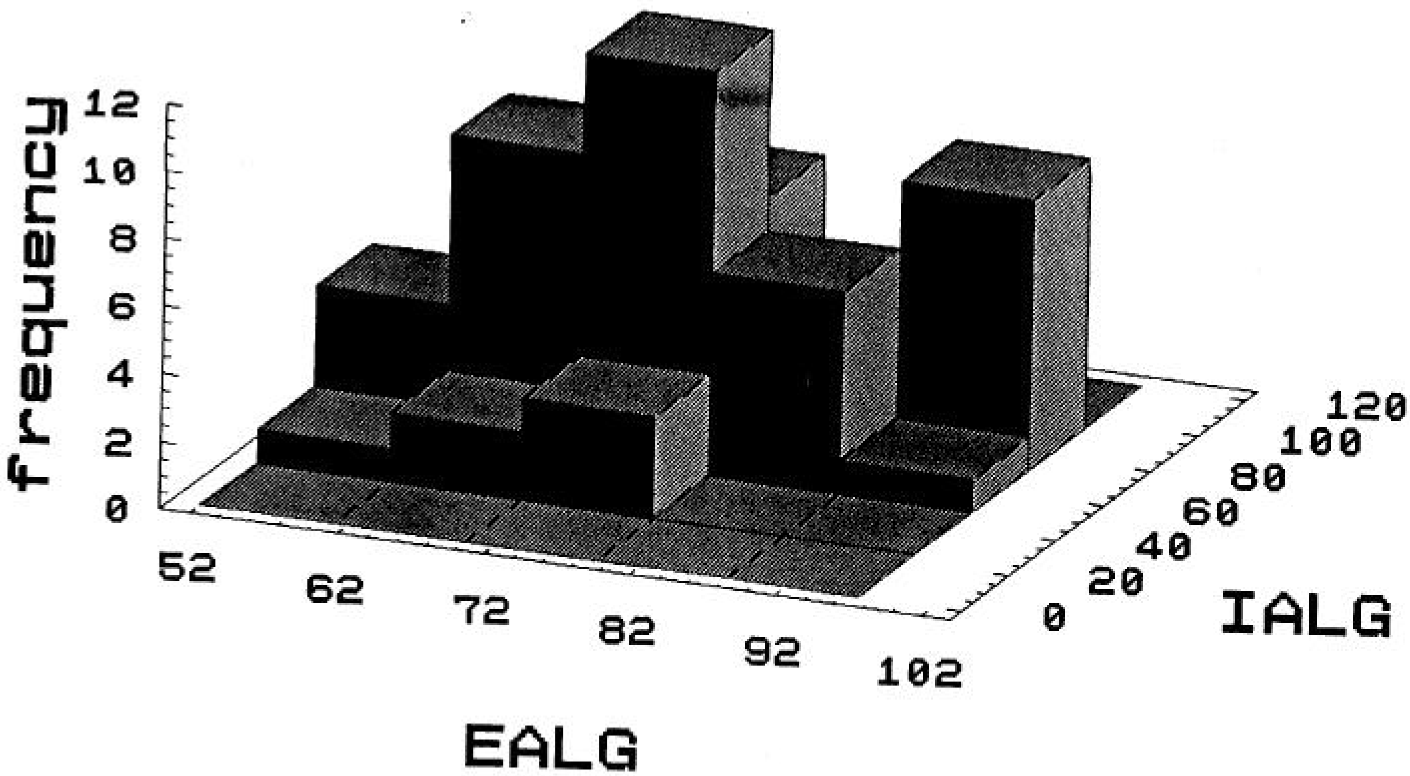
arameter	Estimate	Standard Error	T Value	Prob. Level
ntercept	50.6225	5.75267	8.79983	.00000
lope	0.368555	0.0831701	4.43134	.00004

Analysis of Variance

ource	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
odel	1919.2509	1	1919.2509	19.637	.00004
rror	5864.2491	60	97.7375		
otal (Corr.)	7783.5000	61			

orrelation Coefficient = 0.496568 R-squared = 24.66 percent
tnd. Error of Est. = 9.88623

Three-D Histogram of elementary algebra scores vs intermediate algebra scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: IALG Independent variable: EAL

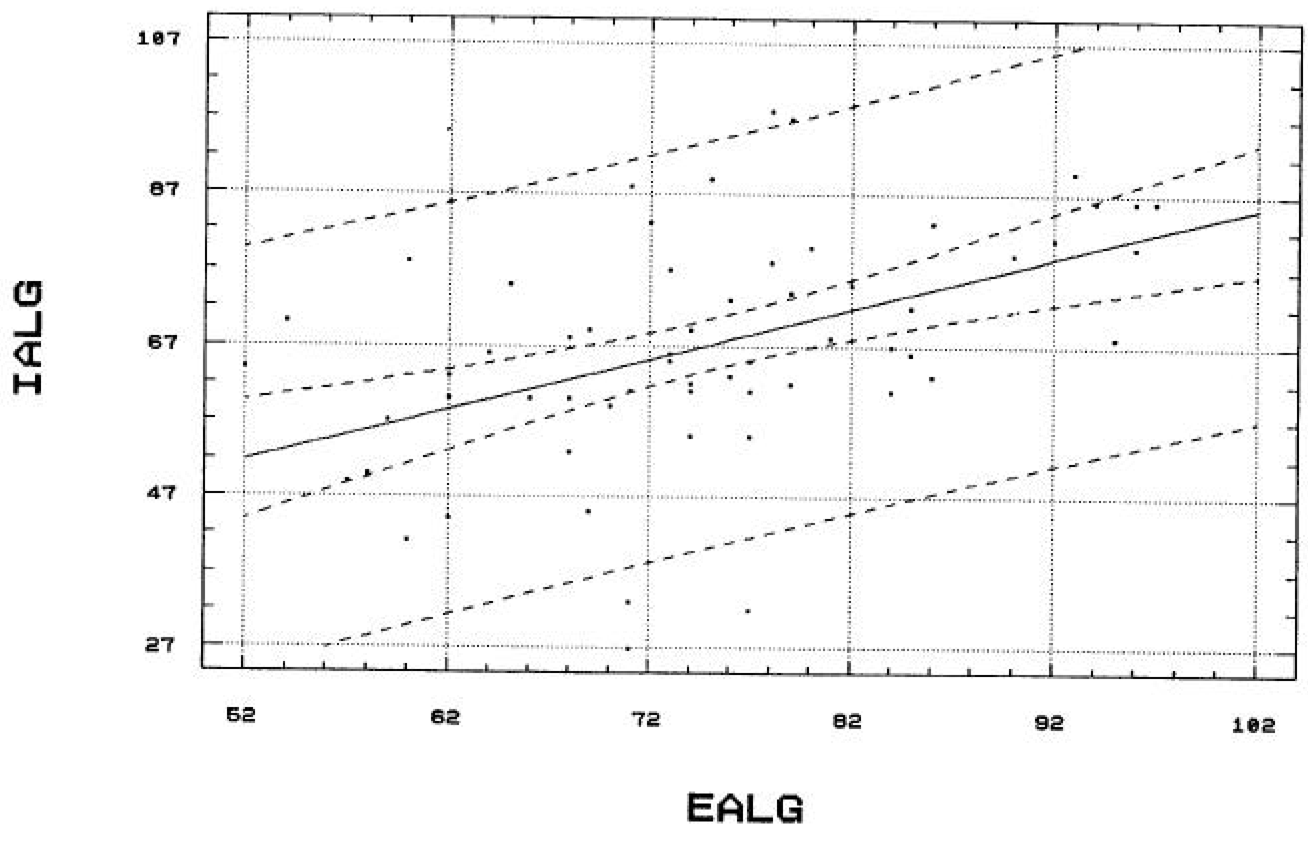
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	16.9872	11.5238	1.4741	.14568
Slope	0.669043	0.15098	4.43134	.00004

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	3484.0440	1	3484.0440	19.637	.00004
Error	10645.456	60	177.424		
Total (Corr.)	14129.500	61			

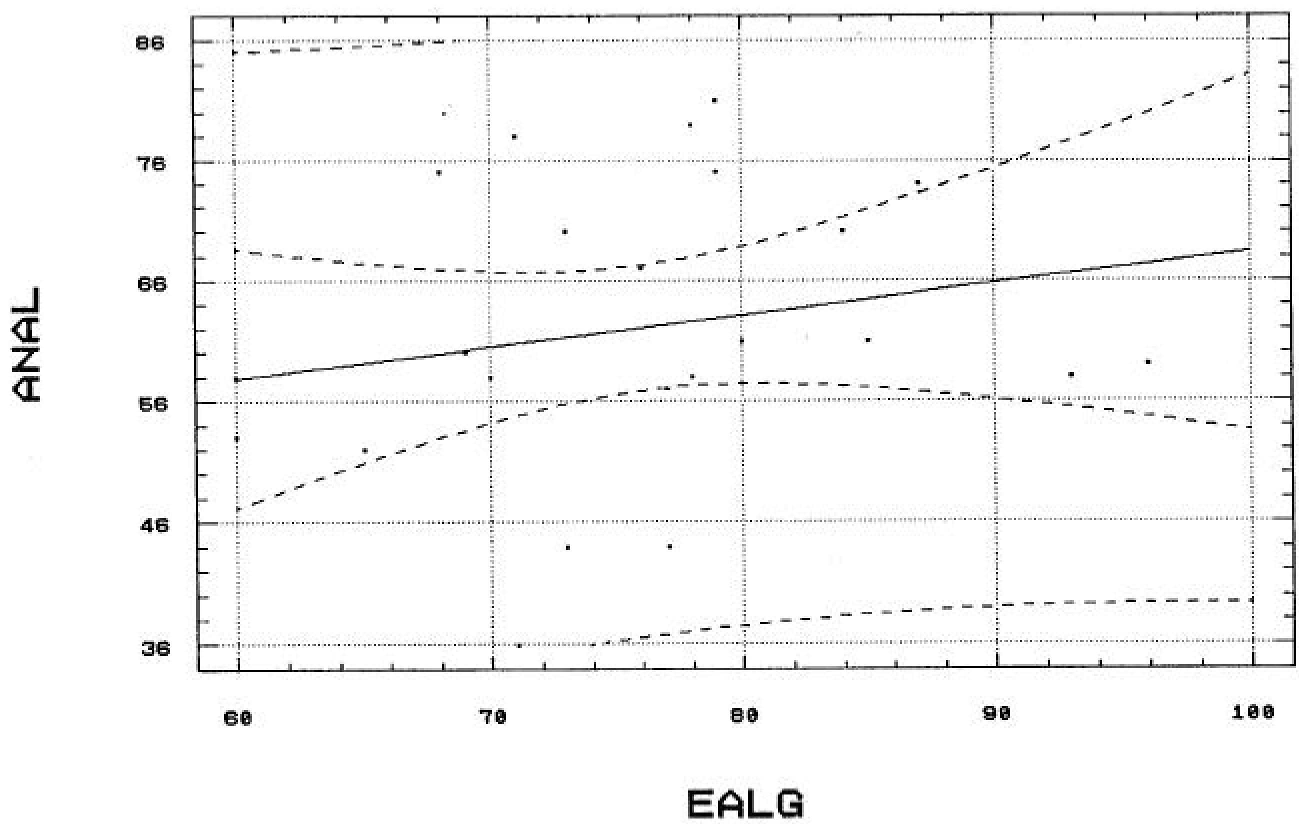
Correlation Coefficient = 0.496568 R-squared = 24.66 percent
Stnd. Error of Est. = 13.3201

Regression of intermediate algebra scores on elementary algebra scores



```
Regression Analysis - Linear model: Y = a+bX
-----------------------------------------------------------------------------------------------
Dependent variable: ANAL                                      Independent variable: EAL
-----------------------------------------------------------------------------------------------
                                  Standard              T                  Prob.
Parameter        Estimate          Error              Value                Level
-----------------------------------------------------------------------------------------------
Intercept         42.1024          21.3621            1.9709               .06206
Slope            0.262804         0.278987          0.941995               .35691
-----------------------------------------------------------------------------------------------

                              Analysis of Variance
-----------------------------------------------------------------------------------------------
Source                 Sum of Squares     Df  Mean Square     F-Ratio  Prob. Level
Model                       127.69986      1    127.69986       .8874       .35691
Error                       3022.1262     21     143.9108
-----------------------------------------------------------------------------------------------
Total (Corr.)               3149.8261     22

Correlation Coefficient = 0.20135             R-squared =    4.05 percent
Stnd. Error of Est. = 11.9963
```

Regression of ANALytic geometry scores on elementary high school algebra scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: EALG   Independent variable: ANA

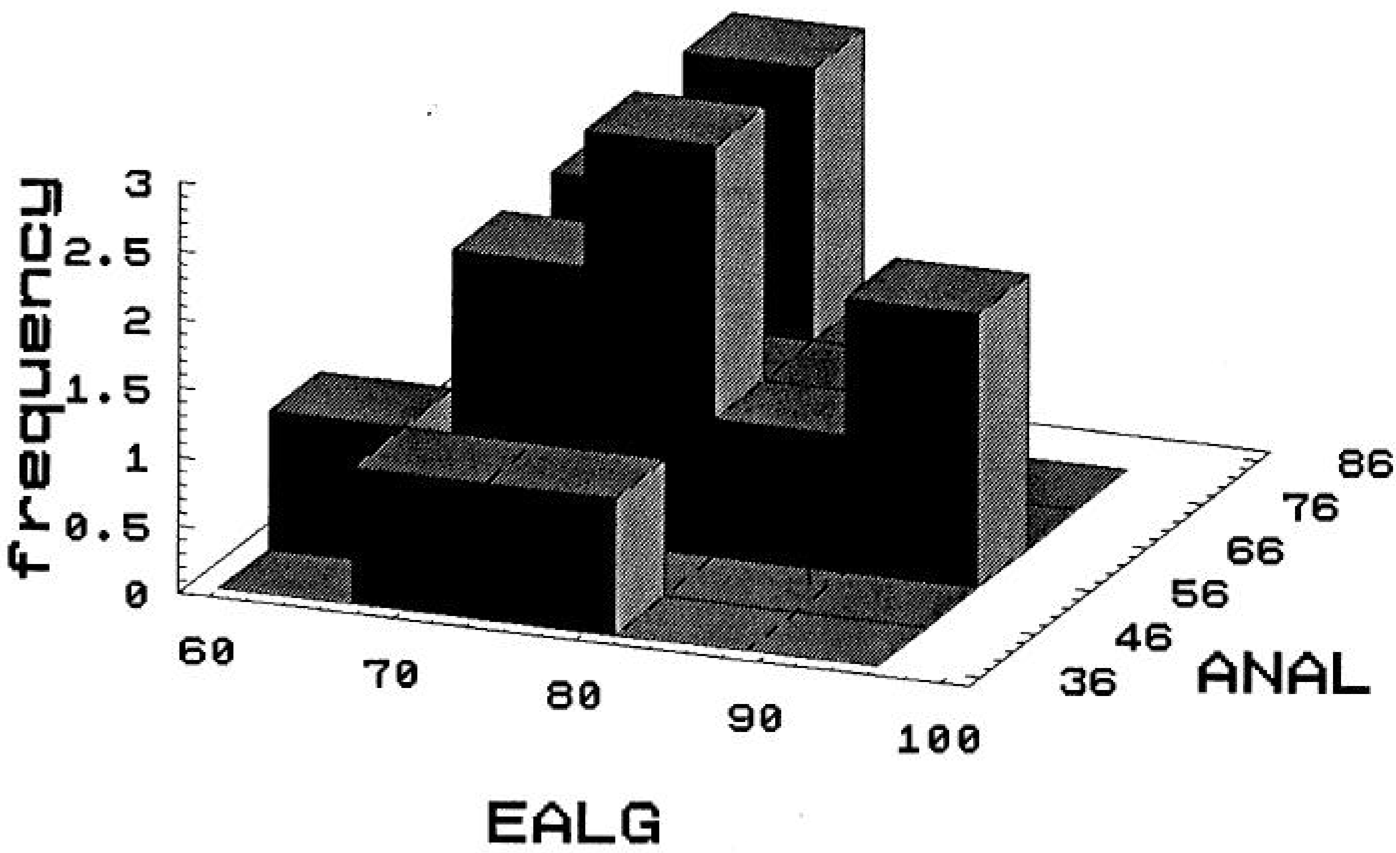
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	66.4655	10.3468	6.4238	.00000
Slope	0.154267	0.163766	0.941995	.35691

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	74.960166	1	74.960166	.88735	.35691
Error	1773.9964	21	84.4760		
Total (Corr.)	1848.9565	22			

Correlation Coefficient = 0.20135   R-squared = 4.05 percent
Stnd. Error of Est. = 9.19108

Three-D Histogram of elementary algebra scores vs analytic geometry scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: TRIG   Independent variable: IAI

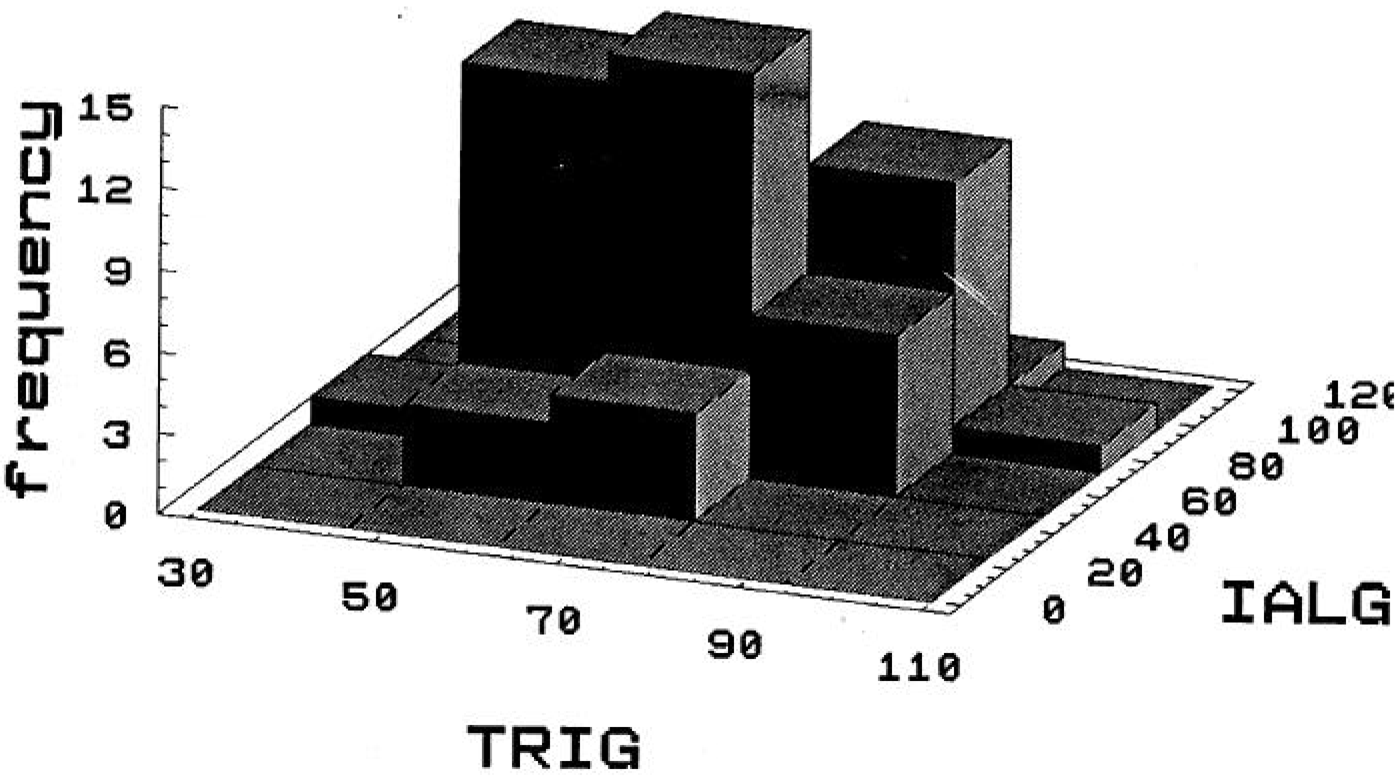
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	38.0339	6.00485	6.33386	.00000
Slope	0.476653	0.0892205	5.34242	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	3055.6484	1	3055.6484	28.541	.00000
Error	6530.6690	61	107.0601		
Total (Corr.)	9586.3175	62			

Correlation Coefficient = 0.56458   R-squared = 31.88 percent
Stnd. Error of Est. = 10.347

Three-D Histogram of trigonometry scores vs intermediate algebra scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: IALG | Independent variable: TRI

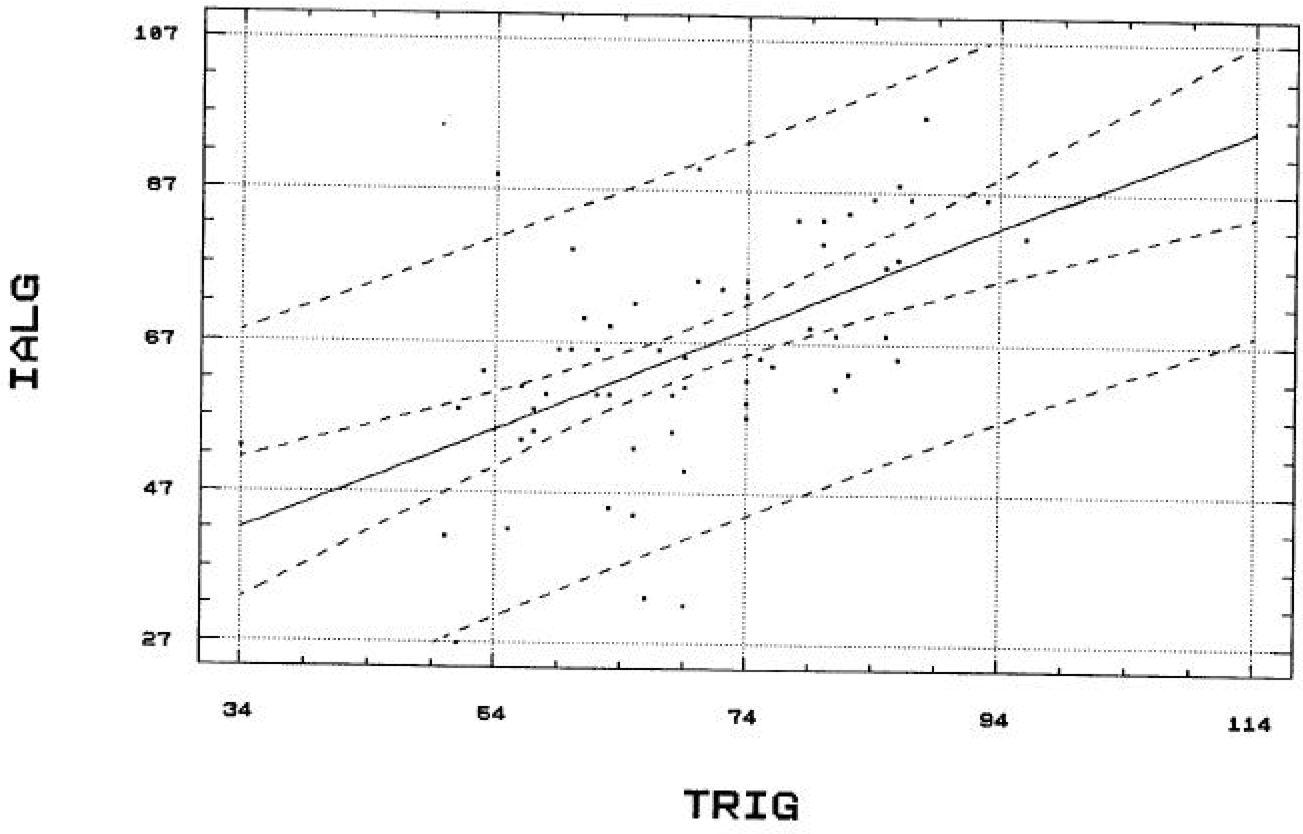
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	19.3227	8.81692	2.19155	.03224
Slope	0.668728	0.125173	5.34242	.00000

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	4286.9684	1	4286.9684	28.541	.00000
Error	9162.3014	61	150.2017		
Total (Corr.)	13449.270	62			

Correlation Coefficient = 0.56458 R-squared = 31.88 percent
Stnd. Error of Est. = 12.2557

Regression of intermediate algebra scores on high school TRIG scores



tegression Analysis - Linear model: Y = a+bX

ependent variable: ANAL   Independent variable: TRIG

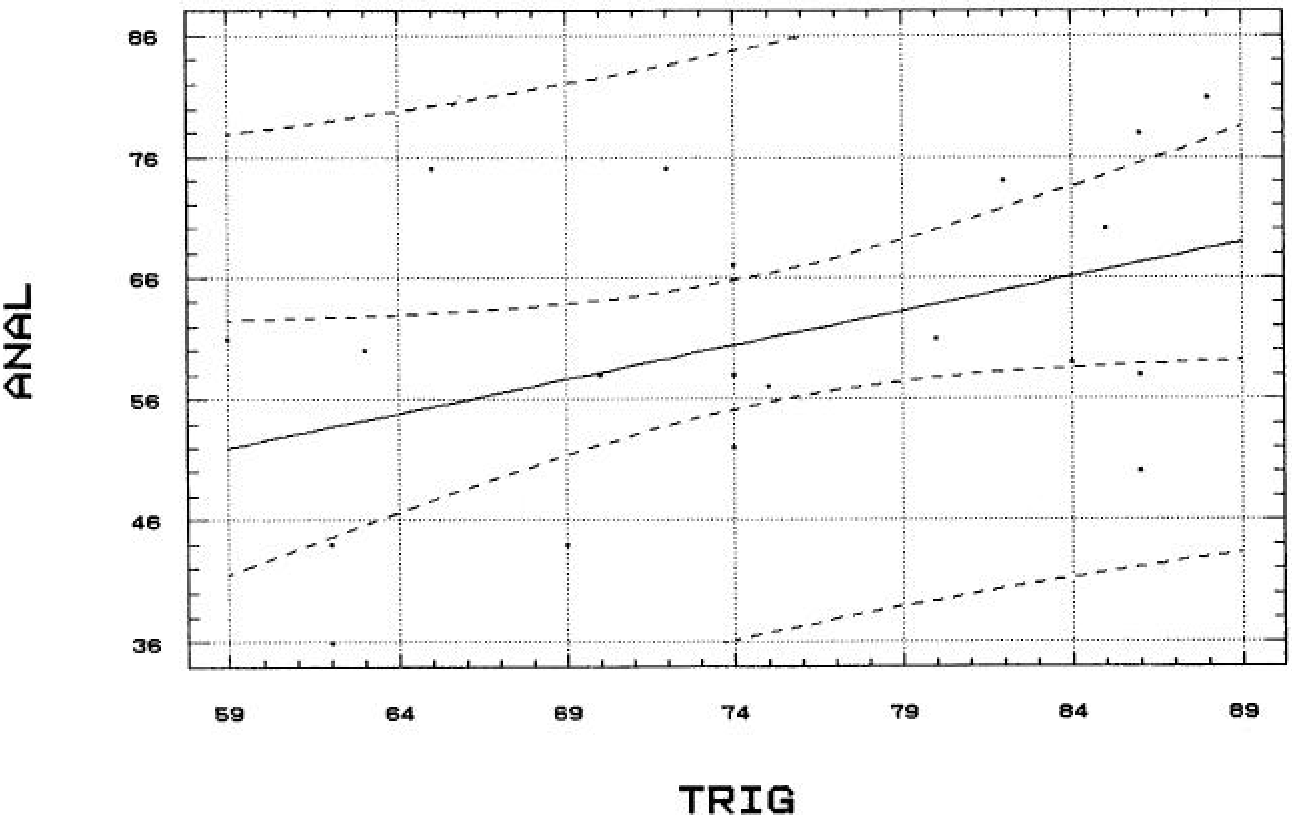
arameter	Estimate	Standard Error	T Value	Prob. Level
ntercept	18.8283	20.602	0.913907	.37285
lope	0.562456	0.273362	2.05755	.05442

Analysis of Variance

ource	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
odel	536.92032	1	536.92032	4.2335	.05442
rror	2282.8797	18	126.8266		
otal (Corr.)	2819.8000	19			

orrelation Coefficient = 0.436361   R-squared = 19.04 percent
tnd. Error of Est. = 11.2617

Regression of ANALytic geometry scores on high school trigonometry scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: TRIG   Independent variable: ANAL

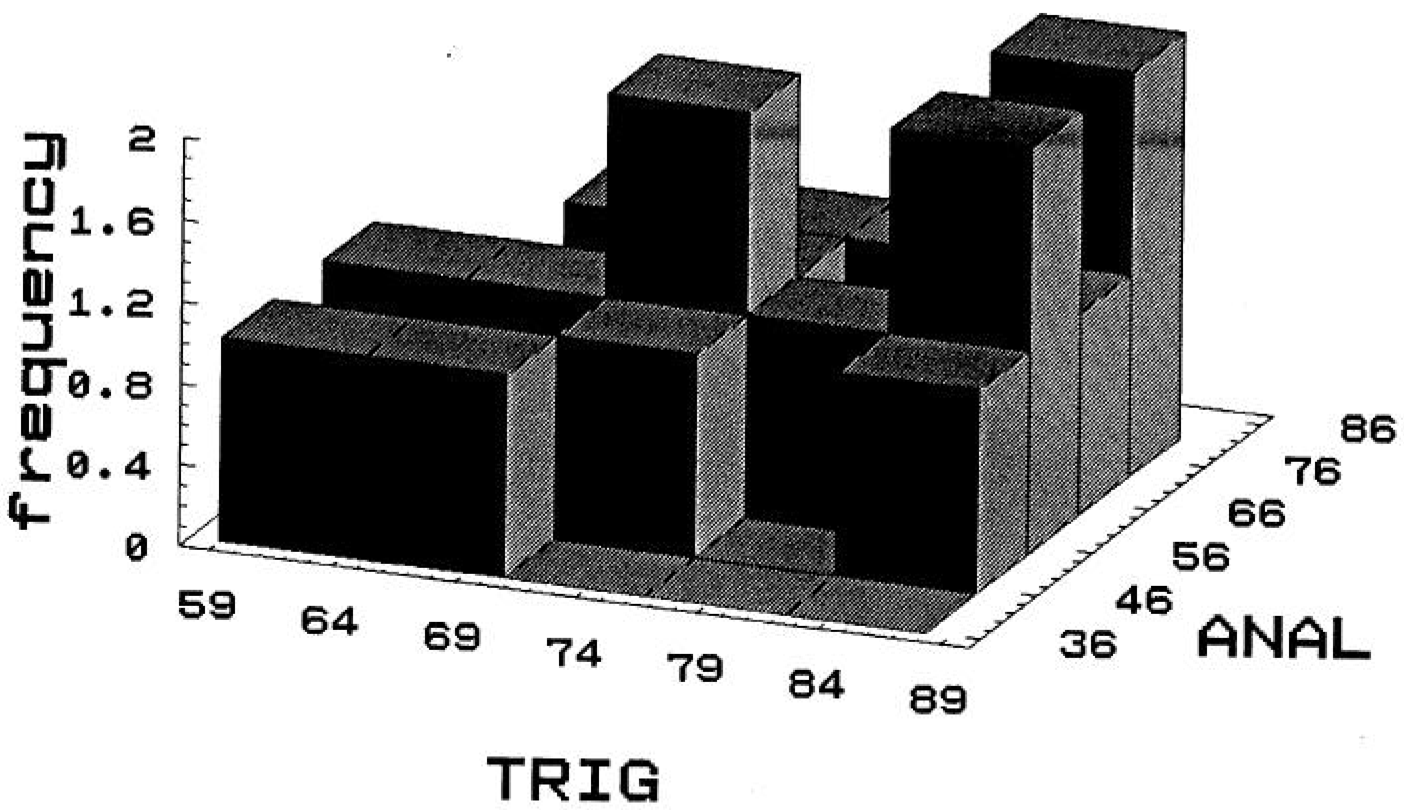
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	54.1832	10.2087	5.30753	.00005
Slope	0.338535	0.164533	2.05755	.05442

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	323.16517	1	323.16517	4.2335	.05442
Error	1374.0348	18	76.3353		
Total (Corr.)	1697.2000	19			

Correlation Coefficient = 0.436361   R-squared = 19.04 percent
Stnd. Error of Est. = 8.73701

Three-D Histogram of trigonometry scores vs high school analytic geometry scores



Regression Analysis - Linear model: Y = a+bX

Dependent variable: IALG   Independent variable: ANA

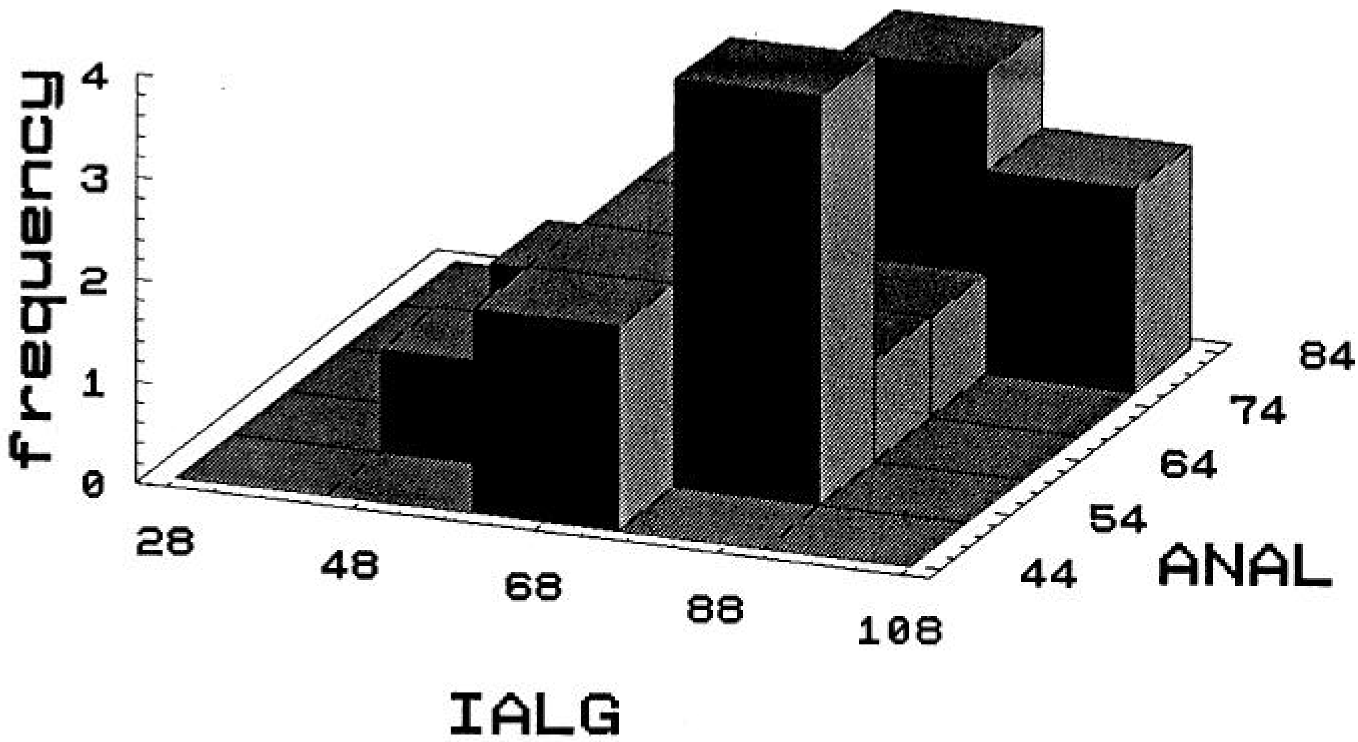
Parameter	Estimate	Standard Error	T Value	Prob. Level
Intercept	32.4069	13.6507	2.37401	.02675
Slope	0.646207	0.211847	3.05034	.00587

Analysis of Variance

Source	Sum of Squares	Df	Mean Square	F-Ratio	Prob. Level
Model	1412.8237	1	1412.8237	9.305	.00587
Error	3340.5097	22	151.8413		
Total (Corr.)	4753.3333	23			

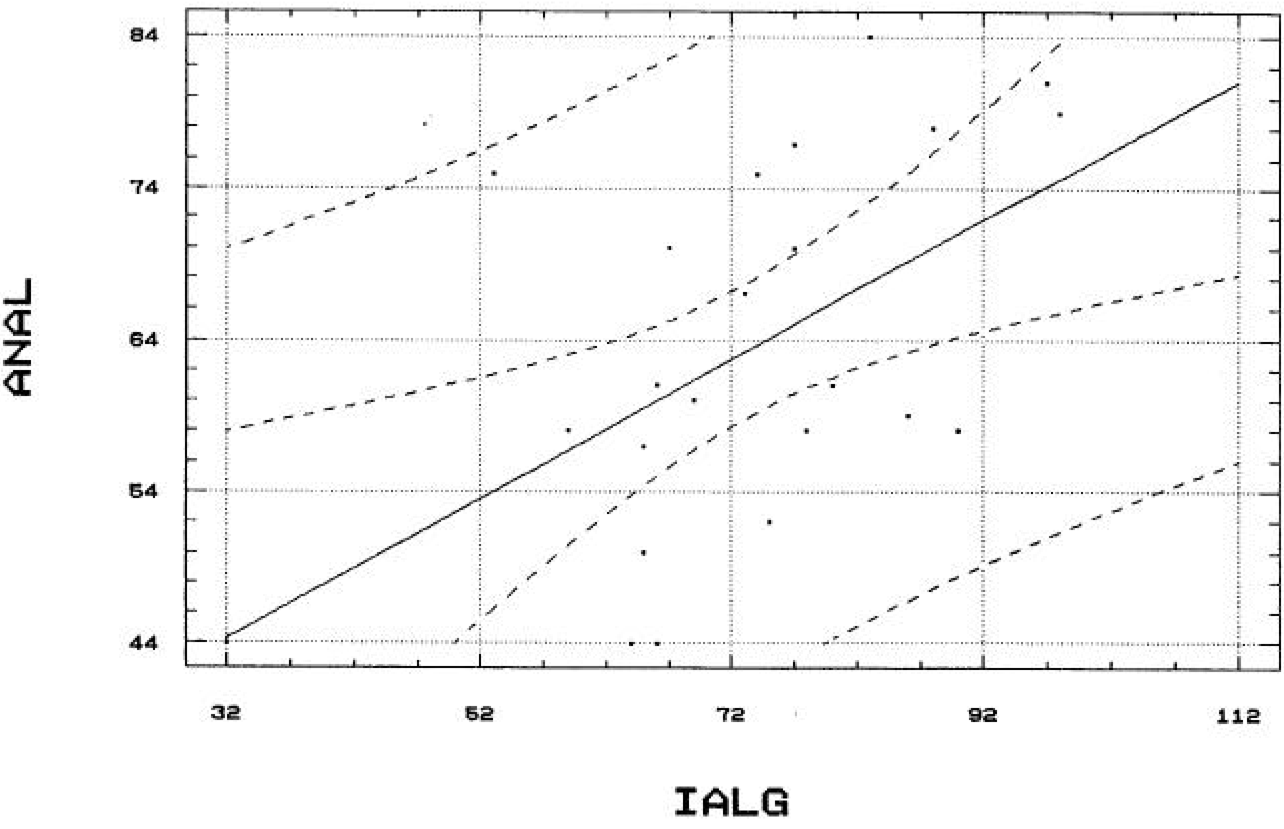
Correlation Coefficient = 0.545186   R-squared = 29.72 percent
Stnd. Error of Est. = 12.3224

Three-D Histogram of intermediate algebra scores vs analytic geometry



```
Regression Analysis - Linear model: Y = a+bX
-----------------------------------------------------------------------------
Dependent variable: ANAL                        Independent variable: IAL
-----------------------------------------------------------------------------
                                Standard              T             Prob.
Parameter        Estimate        Error              Value           Level
-----------------------------------------------------------------------------
Intercept         29.6031        11.2596           2.62914          .01532
Slope            0.459958       0.150789           3.05034          .00587
-----------------------------------------------------------------------------

                           Analysis of Variance
-----------------------------------------------------------------------------
Source                 Sum of Squares     Df  Mean Square    F-Ratio  Prob. Level
Model                       1005.6213      1    1005.6213      9.305       .00587
Error                       2377.7120     22     108.0778
-----------------------------------------------------------------------------
Total (Corr.)               3383.3333     23

Correlation Coefficient = 0.545186              R-squared =  29.72 percent
Stnd. Error of Est. = 10.396
```

Regression of ANALytic geometry scores on scores in intermediate algebra



ELEMENTARY HIGH SCHOOL GEOMETRY IN TERMS OF:

OTIS IQ SCORE	PSAT MATH SCORE	SAT MATH SCORE	ELEM ALGEBRA SCORE	X-AXIS INTERCEPT	# OF CASES	STAND-ARD ERROR	MULT CORR COEFF^2
.3497	.2193			10.2197	101	11.1086	.2391
.2701		.1424		26.4211	101	11.5576	.1027
.3668			.4144	-5.2002	188	10.7582	.342
	.237	.0995		42.4231	72	11.6815	.1357
	.1285		.3764	29.719	98	10.9438	.3031
		.0292	.4051	36.381	101	11.2291	.1961
.2039	.1951	.0499		26.6124	71	11.3064	.1498
.2292	.1083		.3	10.963	97	10.7524	.3137
.1134		.0249	.3468	28.3417	98	11.2027	.179
	.0606	-.1109	.5352	32.8711	70	11.0099	.2603
.1944	.0482	-.128	.4574	18.9605	69	10.8964	.2391

COEFFICIENTS FOR 3, 4 & 5 VARIABLE LINEAR REGRESSION EQUATIONS

TABLE 4

ELEMENTARY HIGH SCHOOL ALGEBRA IN TERMS OF:

OTIS IQ SCORE	PSAT MATH SCORE	SAT MATH SCORE	ELEM GEOM SCORE	X–AXIS INTERCEPT	# OF CASES	STAND-ARD ERROR	MULT CORR COEFF^2
.3725	.428			-3.4135	128	10.5576	.448
.405		.3647		-2.4378	107	10.1451	.4246
.5102			.3821	-12.3905	188	10.3305	.3914
	.3328	.4108		15.6964	74	8.8983	.5608
	.415		.319	19.3716	98	10.0758	.5098
		.4081	.3103	19.8878	101	9.8275	.427
-.0878	.3375	.4437		23.3293	72	8.5609	.5917
.3267	.3512		.2524	-8.3078	97	9.8633	.5332
.3368		.3136	.2528	-7.1284	98	9.5644	.4427
	.264	.3773	.2638	6.3805	70	7.7297	.6543
-.0773	.2815	.403	.2263	14.6532	69	7.6643	.6568

COEFFICIENTS FOR 3, 4 & 5 VARIABLE LINEAR REGRESSION EQUATIONS

TABLE 5

ELEMENTARY HIGH SCHOOL TRIGONOMETRY IN TERMS OF:

OTIS	PSATM	SATM	GEOM	EALG	INTCPT	CASES	S.E.	MLT-R^2
.3155	.1556				17.3702	77	14.327	.0765
.2843		.3123			7.5834	82	14.0955	.1649
.4356			.5256		-20.2313	103	14.3035	.2309
.054				.7057	6.4592	134	13.4211	.3094
	.1275	.2419			39.9559	48	11.8651	.1413
	.1404		.4732		24.3858	54	12.8474	.2344
	-.0341			.6683	17.763	77	13.3411	.273
		.2605	.4639		13.1413	71	12.8914	.2376
		.0917		.616	12.9324	76	13.0268	.3169
			.345	.6769	-8.2385	102	12.4461	.4143
.1261	.106	.2096			29.7101	48	11.9791	.1492
.1662	.1027		.4583		9.2434	54	12.9204	.2409
.1028	-.1052			.64	14.0913	76	13.0292	.2555
.3218		.1692	.4381		-14.5728	70	12.9175	.2565
.0608		.0781		.6104	7.5897	75	13.1627	.3215
-.0388			.3434	.6957	-5.0195	101	12.5517	.4163
	-.0521	.2598	.4977		17.1722	47	10.4336	.3536
	.0201	.0859		.3493	34.3792	47	11.7093	.1907
	-.0561		.3913	.462	10.2063	53	12.2113	.331
		-.0094	.318	.5694	2.5666	70	11.7888	.379
.0912	-.0666	.2359	.4956		9.8803	47	10.5447	.3551
.1392	-.0082	.0442		.3619	22.9263	47	11.8228	.1941
.0289	-.0609		.3894	.4578	7.7031	53	12.3361	.3311
.0862		-.0266	.3096	.5603	-4.587	69	11.9368	.3822
	-.077	.2048	.467	.1245	16.3514	46	10.5637	.3596
.0814	-.0925	.1797	.4643	.1326	9.7863	46	10.6857	.3607

COEFFICIENTS FOR 3, 4, 5, & 6 VARIABLE LINEAR REGRESSION EQUATIONS

TABLE 6

HIGH SCHOOK INTERMEDIATE ALGEBRA IN TERMS OF:

OTIS	PSATM	SATM	GEOM	EALG	TRIG	INTCPT	CASES	S.E.	MLT-R^2
.451	.2657					-2.6489	34	11.9096	.2867
.2817		.3995				2.4533	57	13.0868	.2499
.6836			.3694			-37.3725	62	13.4088	.2484
.5858				.5296		-39.3507	61	12.7588	.3308
.6949					.6294	-57.4498	62	11.1643	.4516
	.2185	.316				27.1232	30	11.2695	.3087
	.2252		.4353			20.3024	34	11.6239	.3205
	.1922			.4739		17.7472	33	11.3323	.3673
	.2916				.5074	9.6366	28	10.5308	.384
		.4427	.3588			5.5138	50	13.1561	.2981
		.3214		.3634		14.1774	50	13.1704	.2966
		.3474			.5652	-1.2583	48	11.4121	.4329
			.3271	.6093		-2.1589	61	13.0471	.3006
			.2355		.6637	2.5121	53	12.8845	.3111
				.4756	.5323	-7.1525	53	12.034	.3993

COEFFICIENTS FOR 3 VARIABLE LINEAR REGRESSION EQUATIONS

TABLE 7

HIGH SCHOOL INTERMEDIATE ALGEBRA IN TERMS OF::

OTIS	PSATM	SATM	GEOM	EALG	TRIG	INTCPT	CASES	S.E.	MLT-R^2
-.0145	.2208	.3194				28.3167	30	12.1902	.3087
.3755	.136		.3949			-12.4601	34	11.5086	.3554
.379	.0975			.4478		-15.8867	33	11.232	.4034
.573	.1777				.449	-41.9442	28	9.8774	.4798
.3208		.3562	.3419			-23.0131	49	13.295	.3131
.3361		.2302		.3601		-16.5286	49	13.2848	.3142
.5134		.1953			.6095	-50.7412	47	11.2046	.4762
.5537			.2154	.5233		-50.9629	60	12.6002	.3693
.6572			.1237		.6001	-59.9607	52	12.0339	.4217
.5981				.3313	.5266	-63.7671	52	11.3644	.4845
	.0434	.3438	.4726			2.9138	30	11.3881	.3967
	.1506	.1614		.3399		18.8416	29	12.0014	.3501
	.2302	.2404			.357	6.6094	25	10.5443	.4677
	.0534		.3941	.4477		2.0157	33	10.9391	.4341
	.2239		.2114		.4755	1.759	28	10.566	.4047
	.1838			.3141	.3941	1.8853	28	10.1639	.4492
		.3101	.2864	.3313		-3.5095	49	13.1066	.3316
		.3353	.1756		.5538	-12.2175	42	12.2162	.4314
		.2443		.2632	.5081	-8.918	43	11.9644	.4409
			.1376	.4845	.4923	-14.9601	52	12.1726	.4096

COEFFICIENTS FOR 4 VARIABLE LINEAR REGRESSION EQUATIONS

TABLE 8

HIGH SCHOOL INTERMEDIATE ALGEBRA IN TERMS OF:

OTIS	PSATM	SATM	GEOM	EALG	TRIG	INTCPT	CASES	S.E.	MLT-R^2
-.0398	.0597	.3532	.4732			6.1552	30	11.6118	.3969
.1693	.1167	.1045		.3759		3.9927	29	12.2197	.3532
.4588	.1655	.1346			.3692	-32.8279	25	10.5334	.4941
.3137	-.0133		.361	.4284		-24.5032	33	10.8916	.4584
.543	.1402		.1357		.4316	-44.2977	28	10.0094	.488
.5183	.0968			.2673	.3582	-43.6156	28	9.6307	.526
.3648		.2052	.2598	.3521		-36.4148	48	13.2152	.3502
.5325		.1937	.141		.5715	-60.2894	41	12.0555	.4744
.5079		.1256		.2094	.5375	-55.096	42	11.8314	.4803
.6175			.0414	.378	.4855	-69.778	51	11.4862	.495
	.0228	.2195	.419	.2603		-.7881	29	11.64	.4131
	.155	.2578	.2174		.3213	-2.2424	25	10.626	.4852
	.1896	.1494		.1903	.345	3.1336	25	10.6847	.4795
	.1089		.2262	.3214	.3574	-6.7209	28	10.1574	.4728
		.2455	.1317	.2505	.487	-16.1645	42	12.2011	.4477
.1136	.0015	.1807	.4144	.2854		-10.5272	29	11.8753	.4145
.4287	.0992	.1572	.1949		.3364	-38.1781	25	10.6572	.5081
.5577	.094	-.0169		.2694	.3544	-46.2429	25	10.5658	.5165
.4819	.051		.1569	.2756	.3353	-46.3881	28	9.7323	.537
.5363		.1089	.0949	.2409	.5045	-64.1118	41	12.0538	.4891
	.1203	.175	.2029	.1707	.3128	-4.7711	25	10.8022	.4946
.5239	.0421	.0145	.1689	.2483	.3274	-49.8278	25	10.7388	.5268

COEFFICIENTS FOR 5, 6 & 7 VARIABLE LINEAR REGRESSION EQUATIONS

TABLE 9

HIGH SCHOOL ANALYTIC GEOMETRY IN TERMS OF:

OTIS	PSATM	SATM	GEOM	EALG	TRIG	IALG	INTCP	CASES	S.E.	M-R^2
.6374	-.229						9.2858	12	11.6245	.1777
.2391		.108					26.6256	21	12.4743	.0411
.2726			.3112				7.8272	25	12.0369	.1113
.3948				.1678			4.2267	23	11.848	.1107
.1863					.5378		-.6741	20	11.4747	.2062
-.1801						.5129	46.465	24	10.5726	.3062
	-.3421	.4695					48.5346	10	13.1102	.0751
	-.3505		1.2214				4.524	12	7.5614	.6521
	-.2173			.1917			66.7985	11	11.1626	.0663
	-.2686				.228		68.1263	9	9.3078	.286
	-.3972					.5009	56.5953	11	10.7526	.3202
		.218	.199				28.5915	19	11.3311	.0936
		.2484		-.0336			43.4475	18	10.9272	.0722
		.1646			.3781		19.54	16	10.2496	.1661
		-.3505				.5825	50.997	20	10.7481	.3006
			.2541	.1769			30.7101	23	12.0644	.0758
			.1363		.5561		9.7015	20	11.5173	.2003
			.1482			.3898	22.986	22	9.9755	.3038
				-.0203	.6961		11.3851	19	11.0248	.2783
				-.1277		.385	44.7073	20	9.6745	.2818
					.0182	.3352	31.6008	18	10.0427	.2512

COEFFICIENTS FOR 3 VARIABLE LINEAR REGRESSION EQUATIONS

TABLE 10

HIGH SCHOOL ANALYTIC GEOMETRY IN TERMS OF:

OTIS	PSATM	SATM	GEOM	EALG	TRIG	IALG	INTCP	CASES	S.E.	M-R^2
.7107	-.5066	.5493					-27.5603	10	13.6252	.1438
-.0265	-.3488		1.2333				6.5371	12	8.018	.6522
.6702	-.2173			-.0102			7.2435	11	10.6597	.255
.3517	-.2985				.2498		29.1789	9	9.7222	.3508
-.0744	-.3995					.5244	63.3938	11	11.4859	.3212
-.0681		.2408	.1976				34.4711	19	11.6944	.0949
-.1139		.2865		-.036			53.3334	18	11.2849	.0765
-.0332		.1767			.3732		22.2202	16	10.6659	.1665
-.2776		-.3236				.6413	76.1656	20	10.95	.3168
.3479			.1824	.1174			.5549	23	12.0377	.1259
.1691			.1146		.5348		-6.5536	20	11.7766	.2131
-.2537			.1486			.4332	46.7415	22	10.0886	.3254
.324				-.0908	.6842		-19.2835	19	11.02	.324
-.1567				-.1364		.435	59.6382	20	9.9165	.2906
-.264					.0143	.4409	58.9714	18	10.2071	.2781
	-.534	.3668	1.5159				-36.5866	10	8.3848	.6757
	.1869	-.3518		-.4048			117.0547	9	11.8334	.1316

COEFFICIENTS FOR 4 VARIABLE LINEAR REGRESSION EQUATIONS

TABLE 11 - PART 1 OF 2

HIGH SCHOOL ANALYTIC GEOMETRY IN TERMS OF:

OTIS	PSATM	SATM	GEOM	EALG	TRIG	IALG	INTCP	CASES	S.E.	M–R^2
	.3783	.337			.2239		46.6374	8	11.0416	.2566
	-.6869	.5507				.5956	21.1059	10	11.9253	.3441
	-.1165		1.442	-.5781			16.4857	11	6.4821	.7245
	-.3476		1.0093		.2697		-.9321	9	8.1643	.5422
	-.3828		1.3405			.0963	-9.3049	11	8.0663	.6652
	-.3229			.1213	.2456		61.3723	9	10.1364	.2943
	-.2675			-.138		.3381	71.5196	10	10.5491	.2617
	-.273				.2166	.0153	68.1728	9	10.195	.2861
		.2356	.1035	-.0502			38.49	18	11.2616	.0803
		.1628	.0113		.3789		18.8531	16	10.6675	.1662
		-.1928	.0864			.4399	40.1317	18	10.6094	.2533
		.1897		-.1445	.3976		27.071	16	10.5788	.18
		-.1301		-.1773		.4154	56.7225	17	10.1866	.2499
		-.0555			.1583	.2369	37.733	15	10.6889	.155
			.138	-.0613	.6968		4.7702	19	11.3083	.2882
			.0903	-.1386		.3739	39.9639	20	9.9401	.2872
			.02		.0799	.3329	30.2455	18	10.3933	.2515
				-.3359	.2006	.3389	48.6743	17	9.8966	.3224

COEFFICIENTS FOR 4 VARIABLE LINEAR REGRESSION EQUATIONS

TABLE 11 - PART 2 OF 2

HIGH SCHOOL ANALYTIC GEOMETRY IN TERMS OF:

OTIS	PSATM	SATM	GEOM	EALG	TRIG	IALG	INTCP	CASES	S.E.	M–R^2
.5249	-.6512	.4281	1.4817				-90.8741	10	8.6431	.7129
.2765	.0666	-.2604		-.3358			80.7218	9	13.1213	.1458
.4886	-.4967	.4117			.2556		-7.23	8	12.2586	.3128
.2631	-.7259	.5757				.5578	-5.3249	10	12.9804	.3524
.1236	-.121		1.3769	-.5805			7.7738	11	6.9367	.7296
.1578	-.3558		.9416		.2767		-13.7706	9	9.0086	.5541
-.0265	-.3836		1.3396			.105	-6.8316	11	8.7108	.6654
.3399	-.3246			.0601	.2578		27.1273	9	10.8532	.3528
.2243	-.2443			-.167		.2574	52.8782	10	11.4503	.2752
1.4495	-.0228				1.1277	-1.0906	-95.7219	9	9.1331	.5417
-.1076		.2721	.1006	-.052			47.9677	18	11.6628	.084
-.0313		.1746	.0086		.3741		21.9259	16	11.1387	.1665
-.4831		-.1159	.0483			.526	84.9268	18	10.5682	.312
-.002		.1904		-.1442	.3973		27.2451	16	11.0491	.18
-.5308		-.0435		-.2272		.5126	106.4031	17	10.0001	.3328
-.4801		-.0604			-.0838	.4442	95.7516	15	10.8646	.2064
.3124			.1103	-.121	.6852		-23.4909	19	11.3539	.3303

COEFFICIENTS FOR 5 VARIABLE LINEAR REGRESSION EQUATIONS

TABLE 12 – PART 1 OF 2

HIGH SCHOOL <u>ANALYTIC GEOMETRY</u> IN TERMS OF:

OTIS	PSATM	SATM	GEOM	EALG	TRIG	IALG	INTCP	CASES	S.E.	M–R^2
-.1584			.0918	-.1476		.4242	54.9723	20	10.2015	.2961
-.2637			.0174		.0159	.4387	57.7594	18	10.5909	.2783
-.1174				-.3503	.1441	.4018	62.8043	17	10.2726	.3261
	-.0666	-.094	1.4654	-.5931			20.722	9	8.5266	.6393
	-.5344	.3902	1.1065		.2723		-29.769	8	10.167	.5273
	-.6335	.4074	1.3403			.2103	-36.4248	10	8.8169	.7012
	-.5722	.5356		.2154	.2548		23.6465	8	12.1197	.2717
	-.1398	-.0902		-.2282		.2629	83.2892	9	12.8564	.1799
	-.2367	.2681			.4217	-.2687	48.2469	8	12.5557	.2791
	-.1424		1.5482	-.5504	.2121		-7.1554	9	8.0819	.6411
	-.1235		1.4951	-.5661		-.0014	12.2608	10	7.6453	.6769
	-.2728		1.2264		.504	-.3036	-16.7072	9	8.5951	.5941
	-.3194			.1329	.2686	-.0287	60.6385	9	11.3288	.2948
		.1843	.0462	-.157	.4025		24.9235	16	11.0381	.1817
		-.1314	.0243	-.1799		.4139	55.4229	17	10.5993	.2504
		-.0485	-.0381		.1554	.2343	40.1678	15	11.2026	.1562
		-.1083		-.3189	.0979	.3603	62.5677	15	10.7876	.2176
			.0886	-.3541	.2088	.3316	43.8125	17	10.2586	.3279

COEFFICIENTS FOR 5 VARIABLE LINEAR REGRESSION EQUATIONS

<u>TABLE 12 – PART 2 OF 2</u>

HIGH SCHOOL <u>ANALYTIC GEOMETRY</u> IN TERMS OF:

OTIS	PSATM	SATM	GEOM	EALG	TRIG	IALG	INTCP	CASES	S.E.	M-R^2
.2308	-.166	-.0188	1.4597	-.5348			-9.229	9	9.7095	.6492
.548	-.6789	.4796	1.1503		.3113		-95.7846	8	11.4015	.6037
.4264	-.6955	.4436	1.3713			.1401	-80.5719	10	9.4935	.7229
.7296	-1.0209	.9229		.5093	.3464		-96.5919	8	14.2632	.3798
.2003	-.2065	-.6404		-.1892		.2464	59.075	9	14.7792	.1872
1.3685	-.1178	.2602			1.163	-1.1497	-103.7884	8	12.59	.5167
.1536	-.151		1.4806	-.5487	.2191		-19.6373	9	9.1844	.6524
.1966	-.1036		1.4898	-.5899		-.0709	-3.8605	10	8.4102	.6872
1.8067	-.0108		1.4846		1.7002	-.1792	-238.8594	9	2.3396	.9774
1.6782	-.163			.3817	1.4208	-1.3916	-143.2161	9	9.7665	.6069
.0115		.1802	.0476	-.1586	.4045		23.8529	16	11.5765	.1817
-.5334		-.0423	-.014	-.226		.5139	107.4005	17	10.4435	.3329
-.5325		-.0412	-.1075		-.1186	.4594	108.9481	15	11.387	.2154
-.5626		-.1217		-.365	-.1946	.621	134.1399	15	10.8565	.2868
-.1134			.0866	-.3677	.154	.3925	57.5746	17	10.6872	.3314
	-.0292	-.1759	1.6077	-.64	.2024		3.9347	8	11.3821	.605
	-.0166	-.133	1.4922	-.626		-.0439	24.8046	9	9.8296	.6405
	-.3282	.2884	1.1756		.5771	-.4097	-32.0891	8	11.759	.5784
	-.4143	.4427		.1821	.4286	-.2425	28.6573	8	15.2644	.2896
	-.095		1.6901	-.5088	.4137	-.2555	-19.9606	9	8.8492	.6773
		-.1145	.0273	-.3264	.0986	.3651	61.4017	15	11.3668	.2182

COEFFICIENTS FOR 6 VARIABLE LINEAR REGRESSION EQUATIONS

<u>TABLE 13</u>

HIGH SCHOOL ANALYTIC GEOMETRY IN TERMS OF:

OTIS	PSATM	SATM	GEOM	EALG	TRIG	IALG	INTCP	CASES	S.E.	M–R^2
.3537	-.295	.0751	1.4648	-.4215	.2514		-50.187	8	15.6258	.6278
.2499	-.0992	-.0711	1.4994	-.5794		-.0659	-5.8815	9	11.8477	.6518
1.8728	-.1935	.284	1.5536		1.6392	-1.6586	-264.7798	8	1.7216	.9955
2.0434	-.8762	1.0595		.8374	1.5602	-1.4637	-268.8709	8	14.4298	.6826
1.7028	.0665		1.7104	-.2641	1.5845	-1.6411	-227.776	9	.7165	.9986
-.5811		-.1123	-.0426	-.3549	-.2053	.6221	138.3157	15	11.5041	.2882
	.4768	-.5458	1.9034	-.8831	.6399	-.624	13.2022	8	13.734	.7125
1.7675	-.0331	.1101	1.6676	-.1843	1.5925	-1.6331	-242.2431	8	0	1

COEFFICIENTS FOR 7 & 8 VARIABLE LINEAR REGRESSION EQUATIONS

TABLE 14

APPENDIX 1 — RAW SCORES

TEST RESULTS OF BOTH BOYS AND GIRLS IN THE TEN SUBJECTS PRESENTED IN THIS PAPER

OTISB	PSATVB	PSATMB	SATVB	SATMB	GEOMB	EALGB	TRIGB	IALGB	ANALB
106					54	28			
112	60	81				69	64		
124	82	94	74	87	72	87	74		
130	99	97	99	92	67	78			
107					26	63			
123					70		86	65	50
95					52	54			
109						54			
103			51	74	45	63			
115					63	66			
108	76	81	67	92	75	82	53		
114					65	63			
114	76	81			64				
109	57	65	70	63	57	66	63	60	
115	84	52	88	64	66	67			
113			91	94	51	76	74	73	67
115					65	62	65	44	
121	78	70	98	92	71	71	86	88	78
116			97	98	86	72		83	
124	90	92	91	91	76	93	70	90	58
120			95	82	74	60		78	58
118	57	88	78	92	82	78		98	79
116	89	62			71	56	16		
114	80	85	80	99	77	79	72	74	75
98	21	73			61	69			
106	71	56				70	59		
115	73	59	94	75	70	70	70		
110	63	45	76	74	67	68	65	53	75
118	92	91	94	84	87	74	79	69	
101	44	65			54	60			53
112	76	76			72	75	74		
113	87	65				34			
120			99	88	56	69	63	69	60
122			89	86	74	74	68	55	
108	89	70			60	63			
107	63	76	65	69	79	81	81	68	
100			46	70	72	37	12		
113					61	62	14		
109					45	58			
120	81	77	90	85	80	83	73		
120	76	91	84	80	77	86	56		
114	49	88	72	90	67		61		
101					72	71	81	61	
101	66	91			67	77	75	65	57
129					80	79	88	97	81
100	21	76	50	63	55	65			
110					60	57			
122	90	99	97	97	64	96	80	80	
107					69	65	54		
98					58	57	51		
99					56	59	54		
116						45	66		
119			81	63	58	75	51		
118			84	97		71	88		
100			55	89	85	70	74	59	58
108	35	50			61	51			
110	87	65				80	60		

OTISB	PSATVB	PSATMB	SATVB	SATMB	GEOMB	EALGB	TRIGB	IALGB	ANALB
118			74	90	54	73	31		
109			80	98	65	79	66		
126			83	99	82	73	62	66	44
115					63	73	52		
118						59	51		
116	68	69	67	89	61	86	53	63	
113			22	94	69	75	54	89	
109			85	69	57				
110					65	69			
103					61	50	56		
108					60	65	52		
113					71	71	62		36
117			74	96	58	78	86	78	58
122					78	81	67		
114						69	57		
109	71	81				82			
135	99	99	99	96	82	97	80		
101					30				
103	24	31			57	52			
117					53	66			
98					40	52			
123			90	98	80	87	82		74
117			87	67	67	60	50	41	
102					29	38			
119							82	84	
121						76	55		
108	65	77	64	83	65	80			
107					94	87	98		
111	89	75	67	65	57	60			
103	68	73	33	40	85	62	58	60	
112	87	97	62	98	72	96	84	86	59
111					56	67			
119			88	87	79	77	57	55	
101	74	75	37	32	77	54	61	70	
124	93	93	84	82	81	82	70	75	
103					65	57			
114			50	52	69	73	69	65	
92			63	38		41			
110	84	75			58	59			
104	63	82			50	61			
94					41	52			
103					60	70			
100					74	71			
105					51	60			
107	87	86	67	76	61	74			
97	58	26			60	50			
105	90	52	92	80	74	73	85	77	70
121			90	82			56	54	
111	78	68	85	70	74	52			
111	60	85	76	89	78	86	80	83	
117					56	58			
124			85	96			59	66	
123			97	91			60	66	
123			86	91	71	68	68	60	
113					68	76	82	63	
112			72	85			60		
121			96	80				77	77

OTISB	PSATVB	PSATMB	SATVB	SATMB	GEOMB	EALGB	TRIGB	IALGB	ANALB
113	44	73	76	94	75	85	59	66	61
118			68	86				83	84
109					56	62			
115					74	70	72		
106	73	38			38	51			
111	73	76	65	80	53	68	52		
117	78	83	84	74		68	59		
108	76	83				77			
106					51	62			
118	98	96	85	94	88	90	60	79	
123	98	99	98	99	90	97	93	86	
118	87	88				63	37		
113			80	63	34	58			
109	87	83				63	44		
111	93	68	87	91	81	95	85	68	
98					42	60			
107					38	60			
106					55	63	65		
98	44	70	53	65	57	71	66	33	
112					66	60	50		
111	60	83	52	74	77	76	63		
101						69	55		
104					56	61			
			82	91	65	85	65	72	
121	93	90	88	89	90	74	74	62	
101					70	65	40		
97						60			
	76	76	53	80	41	55			
108	63	68	53	63		46			
101			51	60					
115	78	65	79	79	18	68	40		
116	85	90	88	91	54	80	62		
117	57	62	60	86	71	68	60		
100	49	38			46	57		49	
103			41	87	70	71	51	27	
105	60	56			58	34			
117	76	70				58	39		
	44	49				57	25		
127	96	95				97	89		
116	76	96				69	37		
121	94	76				68	66		
TOTALS									
152	72	72	76	76	123	142	95	50	22

OTISG	PSATVG	PSATMG	SATVG	SATMG	GEOMG	EALGG	TRIGG	IALGG	ANALG
126	85	64			62	70			
110	59	43				52	63		
115	52	64	68	84	75	71	57		
121	83	67	87	83	77	60	87		
106	55	64			67	60			
101					67	70			
116					56	57			
107	25	72			68	83			
120	91	79	96	94	58	73	73		
85					53	75			
127	89	96	92	98	71	90	65		
103					67	62	53		
123	86	99	82	97	76	93	92		
129	94	99	95	97	76	96	83		
120	83	94			53				
113	65	77	60	87	61			64	44
101			34	74		52	76	64	
113	80	69	91	91	62	65	74	75	52
123			94	98	75	94	87	86	
103			71	50	65	77	69	32	44
120			77	80	67	68		68	
98					40				
123	59	79			68	64	67	66	
122						66			
118	73	86	80	91	81	92	96	81	
115	59	67				65			
108	68	52			63	51			
117					60				
100						61			
109	73	56			64	36			
105	76	48	73	54	67	60			
111	68	38	74	74	63	69			
114					77	69	63	45	
124	94	94	93	97	60	90			
118					70	79	86		
109					64	62	53	63	
117							34	53	
112			70	84	70	80	80	80	61
119	89	99	92	98	83	97		86	
109	62	75	52	71	52	57			
133	96	99	97	99	95	97	86		
111			92	83	73	50	58		
119					74	84		67	70
112					68	55	53		
104					50	70	68		
112	85	69			46	60			
87					58	56			
104	61	96	33	55	67	78			
115	86	88	57	64	72	58	69	50	
109					64	55			
120			68	95	88	86	78	83	
100	74	78	15	38	53	50			
108					56	74	56	61	
99					50	57			
111			76	86	66	64	57		
106	59	69	26	50	63	60			
95					51	55			

OTISG	PSATVG	PSATMG	SATVG	SATMG	GEOMG	EALGG	TRIGG	IALGG	ANALG
103					50	69			
94					30	55			
113	89	88	91	55	77	80			
112	61	89	26	60	67	73			
116	83	90	71	64	65	74			
105					53	56			
108					44	50			
123	83	95	67	47	65	76			
101					55	73			
99	40	72			60	75			
115			88	74			57	58	
115			94	74			51	58	
115							62	60	
112	48	43	75	63	58	59	74	57	
110							55	42	
122	90	96			80	72	82		
116	80	77				61	81		
110	88	77				60			
100	48	72			55	84	69	61	
131	99	96	99	97	79	77		61	
111			70	68		60	31		
116	91	99	91	96	79	89	81		
99					56	68			
116	83	79	64	86	56	59			
123	97	97	94	96	62	79		62	
100	71	60				63	51		
111	44	33			60	64			
106	68	95			58	62	45		
123	92	84	94	84		59			
			84	70	55	50			
123			92	84		70	50		
87						58			
122	73	79	64	84	55	65	55		
	91	79	85	87		62			
125	94	98			63	80			
111						65			
107	55	48			24	56			
109	90	79				69			
120			87	76	59	55			
123	78	93	78	93	73	86	67		
115	80	75				85	56		
117	73	82				60	57		
120	94	98				88	66		
127	93	97				90	64		
119	97	77				72	56		
116	73	79				73	74		
126	81	84				79	51		
120	90	91				81	54		
122	92	96				93	72		
112	71	84				81	62		
89			37	41		55	34		
TOTALS									
106	63	63	47	47	77	99	53	25	5